Financing Intermediate Inputs and Misallocation: Evidence from China's Firm-Level Data

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Abstract

This paper finds the misallocation of intermediate inputs in China's firm-level data, eliminating which increases the average industry's gross output by 4.98%. Evidences of pre-pay frictions and borrowing constraints on intermediates are found. I build these frictions into a standard firm investment model with adjustment costs and borrowing constraints on capital. Counterfactuals show that the borrowing constraint on intermediates and on capital each accounts for 23% of China's gross output misallocation. Value-added gains from reallocating capital, labor and intermediates are smaller than those using the Hsieh and Klenow (2009)'s approach, both in the model and in the data.

JEL Codes: E44, G31, G32, L60, O33, O47

Key Words: Misallocation, Intermediate Inputs, Pre-pay, Borrowing Constraints

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Operating a firm involves purchases and payments for intermediate inputs, the revenue share of which exceeds 50% (Jones, 2011). In many countries, firms finance these payments for several months due to time-consuming production processes and late sales payments. A strand of the corporate finance literature discusses how to optimize working capital to cover these payments (e.g., Jose, Lancaster, and Stevens, 1996; Deloof, 2003).

Although intermediate inputs are a crucial input for production, little on its misallocation has been discussed. In their seminal work, Hsieh and Klenow (2009) find that marginal revenue products of capital and labor are dispersed across firms within finely defined industries in China, India, and the United States. The potential aggregate output gain if these dispersions were removed is quantified as misallocation, suggesting the existence of frictions or policy distortions. Follow-up studies further investigate the specific frictions and distortions primarily on the inputs of capital and labor, but rarely on intermediate inputs.[1]

This paper studies the intermediate input misallocation. Using the Chinese Above-Scale Industrial Enterprise Survey (AIES) dataset 1998-2007, the paper first finds that intermediate inputs are misallocated. If marginal revenue products of intermediate inputs (MRPM) were equalized across firms *alone*, an average 2-digit Chinese Industrial Classification (CIC) industry would increase its gross output by 4.98% and value-added by 20.61%. The magnitudes are close to the gains obtained by reallocating capital alone.

I then find evidence of two frictions on intermediate inputs in the data, i.e., pre-pay and borrowing constraints. Like in other countries, an average 70-day window exists between firms' expenditures on intermediates and the recovery of sales receipts in China. With this pre-pay friction, financial frictions could hinder firms from choosing the optimal amount of intermediate inputs, and induce a dispersion in its marginal revenue products. I identify the financial friction by looking at whether financial development across Chinese provinces decreases the misallocation measure, i.e., the coefficient of vari-

[1]One exception is Boehm and Oberfield (2018). There are studies that explicitly model intermediate inputs as the third factor in productions, e.g., Jones (2011), Bils, Klenow, and Ruane (2017) and Baqaee and Farhi (2020). But few studies consider explicitly whether intermediates are misallocated in the data and what causes there are.

ation in $MRPM$, more for financially vulnerable industries (Rajan and Zingales, 1998; Braun, 2003). Results suggest that a one standard deviation increase in financial development decreases the coefficient of variation by 33 percent of its standard deviation across industries and provinces. Additional decreases by 6 and 2 percent are identified if the industry is one standard deviation lower than average in asset tangibilities and higher in external financial dependences, respectively.

How could the intermediate input frictions interact with capital frictions that are better understood in the literature (e.g., Bartlesman, Haltiwanger, and Scarpetta, 2013; Asker, Collard-Wexler, and De Loecker, 2014; Midrigan and Xu, 2014)? Does a model with intermediate input frictions quantitatively account more for the misallocation in China's manufacturing sector? To answer these questions, I build the intermediate good frictions into a standard partial equilibrium model of firm investment à la Cooper and Haltiwanger (2006), along with capital frictions of borrowing constraints and adjustment costs. The model is for a representative industry. Firms produce differentiated products that aggregate into a CES industry-level bundle. They maximize net present values of future dividends given stochastic productivities and an option to exit and default every period. Intermediate inputs are determined, and a fraction of the cost needs to be pre-paid a period ahead, which increases firms' financing needs in addition to capital expenditures. Competitive intermediaries provide the financings and charge an interest rate that depends on the default probability.

To quantify how much the model can account for the measured misallocation in the data, I calibrate the model to match key moments in AIES. Reallocation exercises in the model simulated data suggest that the benchmark model well captures the AIES data in the magnitude of misallocations. In the model simulated sample of AIES analog, the gross output would increase by 5.37% and value-added by 13.97%, if only marginal revenue products of intermediate inputs were equalized. These numbers are comparable to those aforementioned for an average 2-digit industry in AIES. Meanwhile, if marginal products of capital, labor, and intermediate inputs were all equalized, the gross output would increase by 21.84%, and value-added by 58.87%. This paper calls the percentage of potential gross output gains as the gross output misallocation and value-added gains as the value-added misallocation. Hence, the model accounts for 100% of the gross output misallocation and 65% of the value-added misallocation in AIES.

Further counterfactuals remove subsets of frictions to quantify the misallocation accounted by each friction. In a model with only the one-period time-to-build capital friction, the gross output misallocation is 13.11%, and the value-added misallocation is 43.69%. The dynamic nature of capital with stochastic firm-level productivities contributes the most misallocation, consistent with Asker et al. (2014). Intermediate input and capital frictions account for about half-half of the difference in misallocations between the benchmark model and the model with only the one-period time-to-build friction on capital. The pre-pay and borrowing constraints on intermediate inputs, mostly the latter, account for 62% of the remaining gross output misallocation. This number is 52% for the capital frictions, mostly from the borrowing constraints on capital. This paper hence contributes to the discussion on financial frictions and misallocation (e.g., Buera and Shin, 2013; Midrigan and Xu, 2014; Moll, 2014), and argues that the effect of financial frictions on misallocation doubles when it distorts both capital and intermediate inputs.

The output gains from reallocation exercises throughout the paper are first defined as changes in gross output and then in value-added. This differs from the conventional value-added approach (Hsieh and Klenow, 2009) that subtracts intermediate inputs from the gross output, and models firms as value-added producers. How do the two approaches differ given the same firm-level dataset? With an elasticity of substitution varying from 3 to 7, I find that the value-added approach gives a value-added misallocation 2 to 3 times as large as that obtained under the gross output approach, both in the model and in the data. Further algebra shows that intermediate distortions, if there is any, could amplify dispersions of returns to capital when one mistakenly models firms as value-added producers. With a multiplicative increase of capital share in the value-added production function, this channel substantially increases misallocations. Combined with my finding of counterfactual experiments, the result suggests that the gap could be narrowed between the empirically sizable misallocation and the moderate misallocation in dynamic models of financial frictions previously found in the literature.

This paper first contributes to studies that follow a *direct approach* and investigate what frictions and distortions could rationalize the misallocation in firm-level datasets for many countries, pioneered by Hsieh and Klenow (2009).[2] One extensively studied theme is capital misallocation and its causes.[3] Some argue the importance of dynamic capital and its adjustment costs (Bartlesman et al., 2013; Asker et al., 2014). Others focus on financial frictions, by which the misallocation could be moderate when firms can accumulate net worth and self-finance. This self-financing channel does not undo misallocation when productivities are less persistent (Caselli and Gennaioli, 2013), when there are fixed cost barriers (Midrigan and Xu, 2014) , when the initial state of the economy is badly misallocated (Buera and Shin, 2013) and when borrowing constraints are endogenous and size-dependent (Gopinath, Kalemli-Özcan, Karabarbounis, and Villegas-Sanchez, 2017; Bai, Lu, and Tian, 2018). This paper shows another possibility that financial constraints bind more if there are recurrent financing needs for intermediate inputs.

This paper also contributes to the work that links intermediate inputs to the TFP. Works include an input-output amplifier of distortions via intermediate inputs (Jones, 2011; Bartelme and Gorodnichenko, 2015; Osotimehin and Popov, 2020), working capital constraints on intermediate inputs and the TFP in crisis of emerging economies (Mendoza and Yue, 2012; Pratap and Urrutia, 2012), and legal enforcement frictions on intermediate inputs transactions (Boehm and Oberfield, 2018). This paper complements these studies by applying working capital constraints of intermediate inputs onto the discussions of misallocation.

This paper is lastly but not leastly related to a large literature on misallocation in China. Hsieh and Klenow (2009) first find substantial misallocation in Chinese data. Brandt, Van Biesebroeck, and Zhang (2012) further document a limited input reallocation across firms in China despite of its high TFP growth over 1998-2007. Explanations for misallocation include preferred lendings to the state-owned firms (Song, Storesletten,

[2]See Restuccia and Rogerson (2013), Hopenhayn (2014) and Buera, Kaboski, and Shin (2015), for a comprehensive review.

[3]Other explanations include information frictions (David, Hopenhayn, and Venkateswaran, 2016; David and Venkateswaran, 2019), heterogeneous and non-isoelastic production functions (Haltiwanger, Kulick, and Syverson, 2018; Uras and Wang, 2018), heterogeneous markups (Peters, 2013; Baqaee and Farhi, 2020) and housing bubbles (Miao and Wang, 2014), to name a few.

and Zilibotti, 2011; Brandt, Tombe, and Zhu, 2013), trade and migration costs (Tombe and Zhu, 2019), entry costs (Brandt, Kambourov, and Storesletten, 2018), and financial frictions (Bai et al., 2018), to name a few examples.

The rest of this paper is structured as follows. Section 1 illustrates the distortions in intermediate inputs allocation in AIES, and introduces frictions of pre-pay and borrowing constraints. Section 2 presents the model. Section 3 calibrates the model, computes the misallocation in the model and in the data, implements decomposition exercises of misallocation contributed by each friction, and discusses economic mechanisms. Section 4 concludes.

1 Intermediate Input Misallocation in Data

This section first describes China's firm-level data, in which I document intermediate input misallocation of a magnitude comparable to capital misallocation. I then show suggestive evidence of pre-pay frictions and borrowing constraints on intermediate inputs.

1.1 Data

This paper uses Chinese Above-scale Industrial Enterprise Survey (AIES) data collected by the National Bureau of Statistics (NBS) from 1998 to 2007. The dataset has been extensively studied (see, e.g., Hsieh and Klenow, 2009; Brandt, Van Biesebroeck, and Zhang, 2014; Bai et al., 2018; David and Venkateswaran, 2019). The unit of observation in the data is a firm with unique registration IDs at the State Administration for Industry and Commerce (SAIC). The dataset combines annual firm-level balance sheets, income, and cash flow statements for all state-owned manufacturing firms and non-state-owned ones with sales above 5 million yuan before 2007.[4] During this period, the number of firms in the data increased from 147,690 in 1998 to 304,599 in 2007.

Variables of interest include gross output, book value of capital, employment, wage bill, intermediate input cost, opening year, inventory, account payables and receivables,

[4]The sales threshold became also applicable for state-owned firms starting from 2007. In 2011, the threshold increased to 20 million yuan.

interest expense, total liability, ownership, and industries. Industries are classified by the 1994 version of the 4-digit China Industrial Classification (CIC) codes before 2002 and the 2003 version after. Concordances of the two versions are done at the broader 2-digit CIC levels. Further, the original data are cross-sectional each year, and I match firms over the years to construct a 10-year unbalanced panel. Nominal values of output and inputs in the unbalanced panel are deflated to constant 1998 yuan using the industry-level deflators.[5]

1.2 Misallocation of Intermediate Inputs

This subsection presents evidence of the intermediate input misallocation in China's data. I start describing the conceptual framework similar to that of Hsieh and Klenow (2009), in which firms are now modeled as gross output producers. Aggregate output Y is produced by a Cobb-Douglas production function of output from each industry

$$Y = \Pi_{s=1}^{S} Y_s^{\theta_s} \tag{1}$$

where θ_s represents the expenditure share of industry s, and $\sum_{s=1}^{S} \theta_s = 1$. Y is used for household final consumption C and as the intermediate input in further productions M, i.e., $Y = C + M$.

Output of industry s, Y_s, is produced under monopolistic competitions with an elasticity of substitution σ

$$Y_s = (\sum_{is}^{N_s} Y_{is}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \tag{2}$$

Firm i in industry s produces gross output quantity

$$Y_{is} = exp(\tilde{z}_{is}) K_{is}^{\tilde{\alpha}_k^s} L_{is}^{\tilde{\alpha}_l^s} M_{is}^{1-\tilde{\alpha}_k^s-\tilde{\alpha}_l^s} \tag{3}$$

and revenue

$$\begin{aligned} P_{is}Y_{is} &= P_s Y_s^{\frac{1}{\sigma}} exp(\tilde{z}_{is})^{\frac{\sigma-1}{\sigma}} K_{is}^{\alpha_k^s} L_{is}^{\alpha_l^s} M_{is}^{\alpha_m^s} \\ &= exp(z_{is}) K_{is}^{\alpha_k^s} L_{is}^{\alpha_l^s} M_{is}^{\alpha_m^s} \end{aligned} \tag{4}$$

[5]Codes used for matching and industry-level deflators are borrowed from Brandt et al. (2012). See Appendix for details about how I match firms over the years.

where $\tilde{z}_{is}$ is log quantity productivity, and z_{is} is log revenue productivity.[6] Here I define $z_{is} = logP_sY_s^{\frac{1}{\sigma}} + \tilde{z}_{is}$. K_{is}, L_{is} and M_{is} denote firm-level capital, labor and intermediate inputs. Note that the industry-specific revenue and cost shares are related, i.e., $\alpha_k^s = \tilde{\alpha}_k^s \frac{\sigma-1}{\sigma}$, $\alpha_l^s = \tilde{\alpha}_l^s \frac{\sigma-1}{\sigma}$, and $\alpha_m^s = (1-\tilde{\alpha}_k^s - \tilde{\alpha}_l^s)\frac{\sigma-1}{\sigma}$.

Firms face distortions τ_{is}^y in output, τ_{is}^m in intermediate inputs, τ_{is}^l in labor and τ_{is}^k in capital, i.e.,

$$MRPK_{is} := \alpha_k^s \frac{P_{is}Y_{is}}{K_{is}} = \frac{1+\tau_{is}^k}{1-\tau_{is}^y}R \tag{5}$$

$$MRPL_{is} := \quad \alpha_l^s \frac{P_{is}Y_{is}}{L_{is}} = \frac{1+\tau_{is}^l}{1-\tau_{is}^y}w \tag{6}$$

$$MRPM_{is} := \quad (1-\alpha_k^s-\alpha_l^s)\frac{P_{is}Y_{is}}{M_{is}} = \frac{1+\tau_{is}^m}{1-\tau_{is}^y}P \tag{7}$$

where P_{is} is firm i's output price in industry s, w and R are labor and capital rental prices. Intermediate input price is P, which is also the price for aggregate output. I use sales for $P_{is}Y_{is}$, wage payment for effective units of labor L_{is}, capital stock for K_{is}, and intermediate input for M_{is} in constant 1998 yuan to compute these marginal products.[7] As in most firm-level datasets, AIES does not report prices and quantities of output and inputs.

My benchmark procedure trims the top and the bottom 1% of $TFPR$ and $TFPQ$ distributions across industries for each year. I set α_m^s and α_l^s, as the median intermediate inputs and labor shares within 2-digit industries in the data. Capital shares are set as 0.85 mi-

[6] I slightly abuse the language here, since log revenue productivity, $TFPR$, is defined as

$$TFPR_{is} = log(P_{is}Y_{is}) - \tilde{\alpha}_l^s log(L_{is}) - \tilde{\alpha}_k^s log(K_{is}) - (1-\tilde{\alpha}_k^s - \tilde{\alpha}_l^s)log(M_{is})$$

which is a linear transformation of z_{is} plus an industry-level constant. Note that log quantity productivity, $TFPQ$, is

$$TFPQ_{is} = log(Y_{is}) - \tilde{\alpha}_l^s log(L_{is}) - \tilde{\alpha}_k^s log(K_{is}) - (1-\tilde{\alpha}_k^s - \tilde{\alpha}_l^s)log(M_{is})$$

[7] Capital stock is constructed using a perpetual inventory method, setting the initial year as 1995. For more discussions, see Brandt et al. (2012).

nus the sum of intermediate input and labor shares, implying an elasticity of substitution $\sigma = 6.67$.[8]

Figure 1 plots dispersions of marginal revenue products for intermediate inputs, capital, and labor across firms and industries in the pooled 1998-2007 data. One can see a substantial dispersion of $MRPM_{is}$ in panel (a), although it is smaller than that of capital and labor in panel (b) and (c). One can further see that histograms for non-state-owned firms almost overlap with those for all firms.

A drawback of measuring $MRPM_{is}$ by Equation (7) is, however, the inability to separate intermediate input distortions τ_{is}^m from output distortions τ_{is}^y.[9] To address the concern, I compare the dispersion of the ratio, $\frac{MRPM_{is}}{MRPL_{is}}$, to that of $\frac{MRPK_{is}}{MRPL_{is}}$, which cancel out τ_{is}^y in the numerator and denominator. Panel (d) of Figure 1 shows that the two ratios have a similar dispersion. If product market distortions were the main cause of the dispersion in $MPRM_{is}$, there would be a much smaller variation in $\frac{MRPM_{is}}{MRPL_{is}}$ than that in $\frac{MRPK_{is}}{MRPL_{is}}$, which is not true in the data.

Next, I quantify the intermediate input misallocation by the potential output gains if the dispersion of marginal revenue products were removed. The exercise reallocates one input at a time to equalize its marginal products across firms within 2-digit industries each year, holding the other two inputs constant.[10] The first row of Table 1 shows gross output and value-added gains by this benchmark exercise. By its largest revenue share, the reallocation of intermediate inputs for an average 2-digit industry results in a gross output gain of 4.98%, close to 4.26% from the reallocation of capital. The industry-level value-added thus increases by 20.61% from the reallocation of intermediate inputs, and 17.88% from that of capital. Compared to the labor input, gains from intermediate inputs reallocations are quantitatively more important.

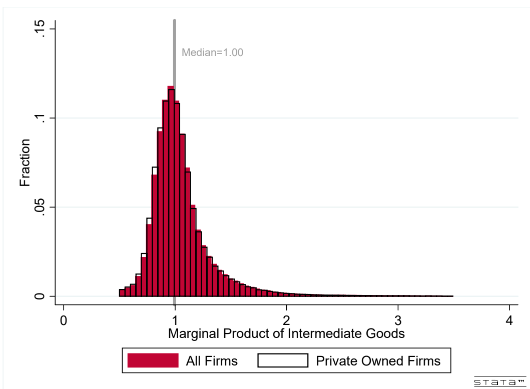
[8]The return-to-scale parameter is thus 0.85 in the revenue production function, consistent with my later calibration results. The implied σ sits in the range of 3 to 10 used in the literature (e.g., Broda and Weinstein, 2010).

[9]For example, studies such as De Loecker and Warzynski (2012) and following papers estimate firm-level markups from the first-order conditions with respect to intermediate inputs. These studies view dispersions in panel (a) of Figure 1 as dispersions of markups τ_{is}^y.

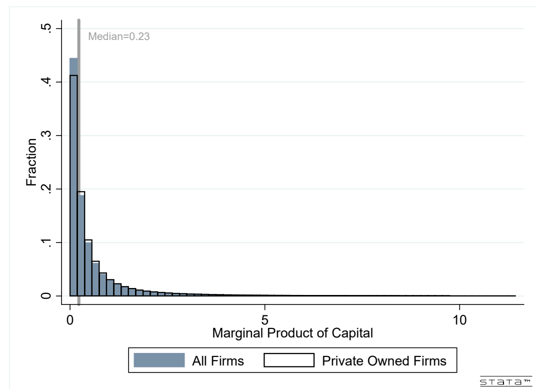
[10]See Appendix for mathematic details.

Figure 1: Marginal Products Histograms, 1998-2007 Pooled Data

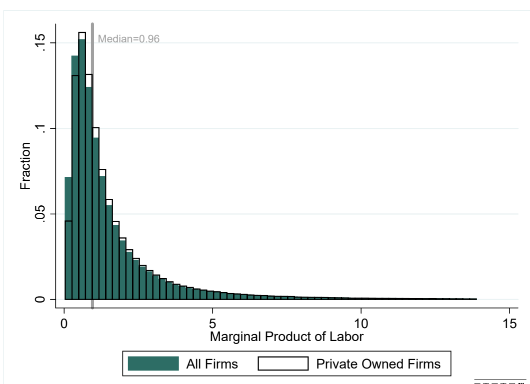
(a) Intermediate inputs



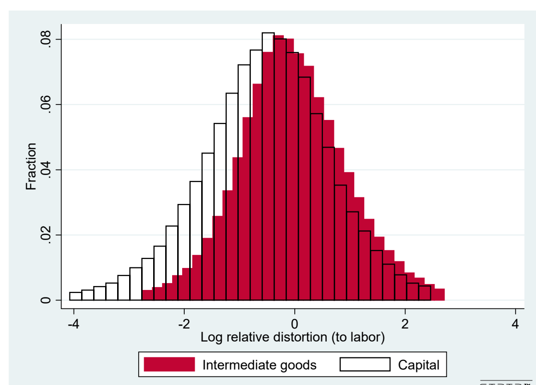
(b) Capital



(c) Labor



(d) Intermediate inputs vs capital



Notes: In panel (a), (b) and (c), solid and hallowed bars plot histograms for marginal revenue products for all firms and non-state-owned (i.e., non-state-owned) firms in 1998-2007 pooled data. Panel (d) plots the ratios of marginal revenue products, $\frac{MRPM_{is}}{MRPL_{is}}$ (solid) and $\frac{MRPK_{is}}{MRPL_{is}}$ (hallowed), for all firms in 1998-2007 pooled data.

Several alternative procedures are done to test the robustness of results. Procedure *Private* keeps only non-state-owned firms for each year. Procedure *4-digit Industry* reallocates each input within 4-digit CIC industries. Procedure *Trim 2.5* trims the top and the bottom 2.5% of $TFPR$ and $TFPQ$ distributions. Procedure *U.S. Shares* uses the industry-level intermediate input and labor shares from the U.S. NBER-CES productivity database.

The next procedure *GE* considers the general equilibrium effect triggered by the intermediate inputs reallocation. After the reallocation, gross output for each industry increases, which expands the supply of intermediate inputs as a CES bundle of outputs from all industries. A lower price of intermediate inputs follows, as well as a higher equilibrium quantity. I approach this idea by setting the revenue shares of intermediate inputs *before* and *after* the reallocation unchanged at the aggregate level, implied by the Cobb-Douglas production function.[11] Numerically, it is to find $\lambda > 1$ such that the reallocation of λ-fold of total intermediate inputs results in an unchanged aggregate intermediate input revenue share.

Lastly, the procedure *Example Industries* checks if the unobserved heterogeneity in intermediate input composition, quantity, and price is the main cause for the measured misallocation. I focus on several industries with the following features: (i) arguably a Leontif combination of different intermediate inputs; (ii) a homogeneous manufacturing process across firms; (iii) competitive and standard, not relation-specific, upstream inputs (Rauch, 1999; Boehm and Oberfield, 2018). These industries are ice making (industry code 1495), cement (industry code 3111), and flat glass (industry code 3141). [12]

The rest of Table 1 presents results under the alternative procedures. One could see that the misallocation of intermediates is robust and not mainly driven by (i) the existence

[11] A more rigorous investigation requires a general equilibrium framework that considers the structure of input-output (Jones, 2011), as well as household labor and saving decisions that affect aggregate labor and capital. See studies of Bartelme and Gorodnichenko (2015) and Hang, Krishna, and Tang (2019) along this line.

[12] Table A1 in the appendix also chooses a set of industries with homogeneous output, i.e., arguably $\tau_{is}^{y} = 0$, as in Foster, Haltiwanger, and Syverson (2008) to illustrate similar magnitudes of intermediate input misallocation. These industries are corrugated & solid fiber boxes, carbon black, ready mixed concrete, plywood, and sugar.

Table 1: Output Gains by Reallocating One Input, 1998-2007 Average

	Gross output gains			Value-added gains		
Procedure	Intermediates	Capital	Labor	Intermediates	Capital	Labor
Benchmark	4.98%	4.26%	2.69%	20.61%	17.88%	11.28%
Private	4.20%	3.49%	2.42%	17.35%	14.63%	10.06%
4-digit Industry	4.50%	3.67%	2.35%	18.64%	15.40%	9.84%
Trim 2.5	3.74%	3.81%	2.57%	16.02%	16.54%	11.14%
U.S. Shares	2.68%	9.22%	7.88%	10.94%	37.91%	32.30%
GE	15.38%	4.26%	2.69%	17.49%	17.88%	11.28%
Example Industries:						
1.Ice making (before 2002)	29.58%	9.67%	5.22%	106.66%	34.77%	19.81%
2.Cement	2.32%	5.46%	2.12%	7.60%	17.50%	6.13%
3.Flat glass	3.28%	1.92%	1.17%	17.87%	10.97%	6.26%

Notes: For the Benchmark case, top and bottom 1% of $TFPR$ and $TFPQ$ are trimmed across industries each year. Reallocation gains are calculated each year at 2-digit industry levels weighted by industry-level gross output shares. Results above report the averages over 1998-2007.

of state-owned firms; (ii) the potentially overestimated intermediate input share in China; (iii) heterogeneity in intermediate input compositions, quantities and prices. The level of misallocation is also higher in terms of gross output, and roughly unchanged in terms of value-added, if the general equilibrium effect is included.

1.3 Intermediate Input Frictions

Next I aim to provide evidence on pre-pay frictions and borrowing constraints of intermediate inputs to account for its measured misallocation.

Pre-pay Using the value-added output measure, the misallocation literature implicitly assumes that intermediate inputs are (i) statically chosen *and* not subject to frictions and distortions; or (ii) perfectly substitutable with the value-added bundle of capital and labor at the firm-level. There has been many discussions that intermediate inputs are imperfect substitutes with the value-added (e.g., Oberfield and Raval, 2014; Ackerberg, Caves, and Frazer, 2015; Gandhi, Navarro, and Rivers, 2017). Nevertheless, the concept of pre-pay is less discussed, and more emphasized in the trade finance literature. Studies (e.g., Ramey, 1989; Petersen and Rajan, 1997; Antras and Foley, 2015) suggest that a time window exists between firms' spending of intermediate inputs purchase and their collection of sales revenue.

This time window is embodied in the notion of cash conversion cycle (CCC)

$$CCC_{is} := \left(\frac{\text{Account Receivables}_{is}}{\text{Sales}_{is}} + \frac{\text{Inventory}_{is}}{\text{COGS}_{is}} - \frac{\text{Account Payables}_{is}}{\text{COGS}_{is}}\right) \times 365 \tag{8}$$

for firm i in industry s where $COGS$ is cost of goods, i.e., the sum of intermediate input costs and wage of production workers. Conceptually, the measure quantifies "the net time interval between actual cash expenditures on a firm's purchase of productive resources and the ultimate recovery of cash receipts from product sales" (Richards and Laughlin, 1980).

Table 2 reports firm-level summary statistics of CCC and its components of days on inventory (DI), days on receivables (DR), and days on payables (DP). On average, firms in AIES receive the cash inflow of sales revenue 70 days, i.e., slightly more than two months, after their cash outflow for intermediate inputs. They hold 71 days of both inventory and deferred payments from their customers, and 56 days deferral of payments to their suppliers. Although there are firms that have a negative CCC, about 90% of them report a positive number.[13]

There is a considerable variation of these measures across firms. The scale of standard deviations in Table 2 suggests that firms differ in converting cash expenditures to cash accruals.[14] Nevertheless, the heterogeneity and its determinants go beyond the scope of this

[13]A different notion is operating cycles (OC), i.e., the sum of inventory and receivables divided by sales standardized by 365 days. The average OC in AIES is 160 days, more than two times of CCC. Thanks to the suggestions of two anonymous referees, I use CCC instead of OC for the coherency with the model section, which abstracts the inter-firm trade credit relationships away.

[14]The pattern is more pronounced when one compares the AIES data to Chinese publicly listed firms, the U.S. Compustat firms and Survey of Small Business Finance (SSBF) firms. For listed firms in China, both average CCC and DI are longer, about 116 days, and 88 days during the same period. The same pattern of longer cycles for large and listed firms could also be found in the United States. These facts may reflect firms' heterogeneous working capital management abilities (Deloof, 2003), or merely different firm sizes and positions in supply chains(Cuñat, 2007).

Table 2: Firm-Level Statistics on Cash Conversion Cycles in AIES Data (Days)

	CCC	DI	DR	DP
Mean	70	71	71	-56
SD	101	88	87	68

Notes: DI, DR and DP are days on inventory, receivables and payables. Statistics of DI and DR are from pooled 1998-2007 data. Statistics of CCC and DP are from pooled 2004-2007 data since AIES does not report firm-level payables before 2004. Top and bottom 1% of each statistic are trimmed.

paper. I thus follow the literature and describe it as part of the production technologies, and constant across firms within industries (e.g., Raddatz, 2003; Tong and Wei, 2011), in order to understand its first-order effect on misallocation.

Financial Frictions With the pre-pay friction, firms could be financially constrained in intermediate inputs when seeking external financings.[15] The constraint is arguably tighter for firms in financially vulnerable industries *ceteris paribus*. Thus these industries may feature higher dispersions in marginal revenue products of intermediate inputs.

To identify the financial friction, I exploit China's provincial differences in the financial market development. I investigate whether there would be a larger drop of intermediate input misallocation for more vulnerable industries than less vulnerable ones, if the industry was in a financially more developed province.[16] The identification strategy is similar to studies such as Raddatz (2003), Tong and Wei (2011) and Manova (2013) that exploit cross-country variations in financial developments.

I use two proxies to represent financial developments across Chinese provinces, $FinDevp_{pt}$. One is provincial fractions of loans lent to non-state-owned firms, $LoanMkt_{pt}$, and the other is the average of $LoanMkt_{pt}$ and provincial fractions of deposits held by non-state-

[15]The same argument applies to the labor input. Focusing on intermediate inputs, rather than labor, is motivated by its substantially larger share in productions.

[16]A more intuitive illustration of financial frictions is Figure A2 that plots measures of intermediate misallocation against financial vulnerable indexes of industries.

owned banks, $FinMkt_{pt}$.[17] Both measures are from the NERI Indexes of Marketization of Chinese Provinces, published by the National Economic Research Institute, China Reform Foundation.[18] I prefer these indexes over private credit to GDP ratios, since the latter may represent an inefficient allocation of credit across provinces rather than the quality of provincial financial markets. For example, during this time period, Beijing had the highest loan-to-GDP ratio (195%) but ranked only 24th in the $LoanMkt$ index and 15th in the $FinMkt$ index among 29 provincial regions with non-missing values.

The identification works via the following regression across industry-province observations:

$$
\begin{aligned}
CV(MRPM_{spt}) = \beta_0 + \beta_1 FinDevp_{pt} + \beta_2 FinDevp_{pt} \times AssetTang_s + \beta_3 FinDep_{pt} \times ExtFin_s + \\
\beta_4 FinDep_{pt} \times CCC_s + \beta_4 SOEShare_{spt} + \beta_5 ExporterShare_{spt} + \sum_{t=1998}^{2007} \delta_t + \sum_p \delta_p + \\
\sum_s \delta_s + \epsilon_{spt} \qquad (9)
\end{aligned}
$$

where s and p are subscripts for 2-digit industries and provinces, respectively. For each industry, $AssetTang_s$, $ExtFin_s$ and CCC_s are asset tangibilities, external finance dependences and cash conversion cycles that are from the Compustat data. Since firm ownerships and exporter statuses may influence misallocation in China's context, I also control for fractions of state-owned firms, $SOEShare_{st}$, and exporter firms, $ExporterShare_{spt}$, for each industry-province cell. δ_t, δ_p and δ_s are time, province and industry fixed effects.

The dependent variable, the dispersion measure in marginal revenue products of intermediate inputs, is its coefficient of variation (CV) at industry-province level, $CV(MRPM_{spt})$. Before calculating CVs, I trim the top and bottom 1% of $MRPM_{ist}$ for each industry each year. The key explanatory variables are $FinDevp_{pt}$, and its interactions with $AssetTang_s$

[17] Non-state-owned banks include joint-equity, city commercial banks, township and village banks, and agricultural cooperatives. They constituted 30% of total assets and outstanding loans, and 27% of deposits of the whole banking sector by the end of 2007.

[18] The foundation is supervised by the National Development and Reform Commission (*fagaiwei*). The NERI indexes were initially compiled in 1997, with an update every three years. It covers five aspects of marketizations of provincial economies: relations between the government and markets, developments of the non-state-owned economies, product markets, factor markets, legal and accounting service markets.

and $ExtFin_s$. If financial frictions are empirically important, one should expect that financially vulnerable industries benefit disproportionately more from the provincial financial market development.

Table 3 presents regression results of which standard errors are clustered within each industry-province cell. I drop cells that have fewer than 20 firms to avoid small sample biases. Columns (1) to (6) use the $LoanMkt$ index as the proxy for provincial financial development levels, while columns (7) to (12) use the $FinMkt$ index. Given the development index, the first five columns use the $CV(MRPM_{spt})$ as the dependent variable, and the 6th column uses the $CV(MRPK_{spt})$ as a comparison. Overall, the table shows that financial market development decreases misallocations of intermediate inputs and of capital, especially for financially vulnerable industries. For instance, if the financial development index increases by 1 S.D. in column (1), $CV(MRPM_{spt})$ decreases by 0.039 for an average industry, and by 0.007 additionally if the industry has an asset tangibility measure 1 S.D. lower than the average. Similarly, for an industry that has external finance dependence 1 S.D. higher than the average, there is a further drop in $CV(MRPM_{spt})$ by 0.002.[19] These numbers are fairly stable across specifications except for column (4) and (10), and the effect via the asset tangibility measure is statistically more significant.

Is the result economically significant? If one looks at statistics of $CV(MRPM_{spt})$ across industry-province cells, it averages 0.2354 with a standard deviation of 0.1182. Hence, when the financial market index, e.g., $LoanMkt$, increases by 1 S.D. in column (1), the decrease of intermediate inputs misallocation coming from this channel is about 33 percent of the standard deviation of $CV(MRPM_{spt})$. This number is 39 percent for industries with 1 S.D. asset tangibility lower and statistically significant. These results suggest that financial frictions play a role in inducing intermediate input misallocation.

[19]Consider the 1 S.D. increase of financial development index, $LoanMkt$, as relocating an industry from Tibet to Fujian province. Further, consider the 1 S.D. tangibility decrease as shifting from industry 31 *non-metal mineral products* to industry 23 *printing and publishing*, and 1 S.D. external finance dependence increase as shifting from industry 17 *manufacture of textiles* to industry 39 *electronic equipment and machinery*.

Table 3: Effects of Provincial Financial Development on Misallocation for Industries with Different Financial Vulnerabilities, Industry-Province Clustered

	$FinDevp = LoanMkt$						$FinDevp = FinMkt$					
	CV(MRPM)					CV(MRPK)	CV(MRPM)					CV(MRPK)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$FinDevp_{pt}$	-0.0120**	-0.0117**	-0.0131**	-0.0012	-0.0103	-0.0276	-0.0130*	-0.0130*	-0.0134*	0.0018	-0.0117	-0.0133
	(0.0040)	(0.0048)	(0.0029)	(0.6357)	(0.0504)	(0.1511)	(0.0113)	(0.0117)	(0.0118)	(0.5882)	(0.0657)	(0.6553)
$FinDevp_{pt} \times AssetTang_s$	0.0176**	0.0172**	0.0124	-0.0005	0.0154*	0.0520	0.0182**	0.0180**	0.0165*	-0.0067	0.0161	0.0385
	(0.0015)	(0.0019)	(0.0513)	(0.8783)	(0.0234)	(0.0660)	(0.0087)	(0.0091)	(0.0362)	(0.1305)	(0.0546)	(0.3735)
$FinDevp_{pt} \times ExtDep_s$	-0.0024	-0.0022	-0.0026	-0.0034*	-0.0023	-0.0192*	-0.0032	-0.0031	-0.0033	-0.0043*	-0.0032	-0.0219*
	(0.1152)	(0.1397)	(0.0839)	(0.0369)	(0.1201)	(0.0103)	(0.1206)	(0.1313)	(0.1115)	(0.0351)	(0.1231)	(0.0380)
$FinDevp_{pt} \times CCC_s$	0.0111***	0.0110***	0.0097***		0.0108***	0.0172	0.0149***	0.0148***	0.0144***		0.0146***	0.0146
	(0.0000)	(0.0000)	(0.0000)		(0.0000)	(0.1267)	(0.0000)	(0.0000)	(0.0000)		(0.0000)	(0.3965)
$CV(MRPK_{spt})$		0.0079*						0.0043				
		(0.0167)						(0.1511)				
$FinDevp_{pt} \times Upstream_s$			0.0089						0.0029			
			(0.1612)						(0.6947)			
$FinDevp_{pt} \times CCC\_SSBF_s$				0.0122***						0.0156***		
				(0.0003)						(0.0002)		
$FinDevp_{pt} \times CCC\_LST_s$					-0.0007						-0.0007	
					(0.5770)						(0.6793)	
$SOEShare_{spt}$	0.0459**	0.0439**	0.0467**	0.0483**	0.0454**	0.2508**	0.0299*	0.0288*	0.0301*	0.0323*	0.0301*	0.2550**
	(0.0017)	(0.0025)	(0.0014)	(0.0011)	(0.0019)	(0.0029)	(0.0391)	(0.0455)	(0.0386)	(0.0264)	(0.0383)	(0.0046)
$ExporterShare_{spt}$	0.0324*	0.0305*	0.0342**	0.0342**	0.0307*	0.2443***	0.0230*	0.0219	0.0234*	0.0236*	0.0213	0.2479***
	(0.0105)	(0.0153)	(0.0073)	(0.0073)	(0.0157)	(0.0002)	(0.0487)	(0.0592)	(0.0460)	(0.0436)	(0.0673)	(0.0004)
Constant	0.3382***	0.3249***	0.3428***	0.3374***	0.3385***	1.6834***	0.2758***	0.2687***	0.2772***	0.2744***	0.2776***	1.6369***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Year FE	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Province FE	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Industry FE	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
N	6669	6669	6669	6669	6640	6669	5999	5999	5999	5999	5975	5999
Adj. R-sq	0.446	0.447	0.447	0.446	0.447	0.255	0.449	0.449	0.449	0.447	0.450	0.257

Note: $FinMkt_{pt}$ index starts from 1999 and hence the number of observations drops when the $FinDevp_{pt}$ is proxied by $FinMkt_{pt}$. Industry-province cells with fewer than 20 firms are dropped. P-values in parentheses are 0.05 for *, 0.01 for **, and 0.001 for ***.

I also include the interaction term between $FinDevp_{pt}$ and CCC_s in the regression. Coefficient estimates suggest that industries with longer CCCs benefit less from the financial market development in lowering the intermediate input misallocation. This is in contrast to the view that these industries are more vulnerable in working capital financings and hence should benefit more. However, one should also note that industries with longer $CCCs$ feature more quasi-fixed intermediate inputs. To further make sure that this result is not driven by large firm sizes in Compustat, I include alternative specifications that uses CCC measures from SSBF, CCC_SSBF_s, and that controls for the industry upstream index, $Upstream_s$, in case firms in different supply chain positions may issue trade credit differently.[20] I also include a specification that further controls for CCC measures of publicly listed Chinese firms, CCC_LST_s. The positive coefficient for the interaction term yet stays robust across specifications.

The last set of results in Table 3 are as follows. First, one may wonder whether the intermediate input misallocation is simply mirroring the capital misallocation. This is not the case. Controlling for the capital misallocation, i.e., $CV(MRPK_{spt})$, coefficients of $FinDevp_{pt} \times AssetTang_s$ and $FinDevp_{pt} \times ExFindep_s$ in column (2) and (8) barely change, compared to column (1) and (7). Second, similar results are found that financial market development decreases misallocation of capital more for financially vulnerable industries, although the effect via the external finance dependence measure is more significant for the case of capital.

To summarize, Section 1 presents the misallocation of intermediate inputs in AIES. Reallocating intermediate inputs alone generates sizable gross output and value-added gains, comparable with the gains from reallocating capital. This section also provides suggestive evidence on two intermediate input frictions: pre-pay and borrowing constraints.

[20] The upstream index for each industry is calculated by dividing the distance to final demand, D, by the sum of D and the number of earlier production stages, N, using the U.S. input-output table (Fally, 2012; Antràs, Chor, Fally, and Hillberry, 2012).

2 Model

If there are borrowing constraints on both intermediate inputs and capital, constrained firms may face a trade-off between the two inputs, as empirically documented in Fazzari and Petersen (1993). This section incorporates the pre-pay friction and borrowing constraints on intermediate inputs into a standard partial equilibrium firm investment model à la Cooper and Haltiwanger (2006), to quantify their roles in accounting for misallocation separately from the roles of capital frictions. I model two capital frictions here, i.e., borrowing constraints and adjustment costs, which are well studied in the literature.

2.1 Firms

The infinite-horizon economy is populated with a mass M_t of heterogeneous firms at time t that grows over time. Given an exogenous productivity level, a firm is a decreasing-return-to-scale technology that produces gross output with inputs of intermediates, capital, and labor.[21] The revenue production function for firm i at time t is

$$p_{it}y_{it} = exp(z_{it})k_{it}^{\alpha_k}l_{it}^{\alpha_l}m_{it}^{\alpha_m} \tag{10}$$

where $p_{it}y_{it}$ is the gross output revenue, z_{it} is the log revenue productivity, and k_{it}, l_{it}, and m_{it} are capital, labor, and intermediate inputs with their revenue shares α_k, α_l, and α_m. The sum of revenue shares equals to $\eta = 1 - 1/\sigma$, where σ is the elasticity of substitution in the CES aggregation within the representative industry.

Firm-level productivity z_{it} has a permanent firm-level component $\bar{z}_i$, $\bar{z}_i \sim N(\mu_{\bar{z}}, \sigma_{\bar{z}}^2)$ and a transitory component μ_{it}, where μ_{it} follows an AR(1) process with persistence ρ and a shock term $\epsilon_{it+1} \sim N(0, \sigma_\epsilon^2)$:

$$\mu_{it+1} = \rho\mu_{it} + \epsilon_{it+1} \tag{11}$$

Given productivity z_{it}, capital k_{it} and intermediate inputs m_{it}, firms choose labor input

[21] In the model, firms are modeled as revenue producers in a representative industry. I will convert output gains into quantity in the later model-simulated data using the same conceptual framework as in Section 1.

l_{it} to maximize their gross output net of labor payment, π_{it}:

$$\pi_{it} = \max_{l_{it}} p_{it} y_t(z_{it}, k_{it}, m_{it}, l_{it}) - w l_{it} \tag{12}$$

where p_{it} is the output price and w is the wage. Labor input is separable from other inputs by its intra-period flexibility without frictions.

Pre-pay Firms pre-pay for intermediate inputs m_{it+1} one period in advance, when choosing next period capital k_{it+1}. For m_{it+1} intermediate inputs, firms pay ω fraction, ωm_{it+1} at time t, and the remaining $(1-\omega)m_{it+1}$ at time $t+1$. [22] In this environment, firms need working capital to pay for intermediate inputs before sales revenue is collected. If the realization of the next period productivity z_{it+1} is relatively low, the pre-paid level m_{it+1} could be too high. In this case, firms can choose $\tilde{m}_{it+1} < m_{it+1}$ to maximize profit at time $t+1$ by selling off the extra intermediate good $m_{it+1} - \tilde{m}_{it+1}$.[23] However, if the pre-paid intermediate inputs level m_{t+1} is too low to be optimal in a high productivity realization z_{it+1}, firms cannot adjust the intermediate inputs beyond m_{it+1}. In other words, firms choose $\tilde{m}_{it+1} \leq m_{it+1}$ to maximize the profit Π_{it+1} after the payment for intermediate inputs

$$\Pi_{it+1} = \max_{\tilde{m}_{it+1} \leq m_{it+1}} \pi_{t+1}(z_{it+1}, k_{it+1}, \tilde{m}_{it+1}) - (1-\omega)m_{it+1} + (m_{it+1} - \tilde{m}_{it+1}) \tag{13}$$

Firms also adjust their capital stock given fixed and convex adjustment cost. If $k_{it+1} \neq k_{it}$, adjustment cost $C(k_{it}, k_{it+1}) = \xi k_{it} + \frac{\theta(k_{it} - k_{it+1})^2}{2k_{it}}$. If on the other hand, firms choose $k_{it+1} = k_{it}$, adjustment cost $C(k_{it}, k_{it+1}) = 0$.

Borrowing Constraints Firms save and borrow at financial intermediaries. The sav-

[22] The time arrangement could be misleading if one interprets that another set of firms receive and benefit from these early payments. What ω means is the *net* time window between cash receipts of sales and cash expenditures. For example, ω could be 1/6 if a 2-month inventory stock works best for productions, and even if cash payments to the suppliers and from the customers happen right after the transactions. I choose the time arrangement to conveniently introduce the need of working capital.

[23] Equivalently I assume that there is no (inter-temporal from t to $t+2$) inventory of intermediates, and if there is, the inventory holding cost is infinitely high. I follow this approach to reduce the dimensionality to lower computational costs. See Appendix on an inventory model and discussions on why quantitatively it may not matter.

ing interest rate is r_1. When firms borrow, they issue one-period corporate bonds. As detailed later in the section of financial intermediaries, the price of corporate bonds, $q_{it}(z_{it}, b_{it+1}, k_{it+1}, m_{it+1})$, depends on firms' fundamentals: current productivity z_{it}, future debt b_{it+1}, future capital stock k_{it+1} and future intermediate inputs m_{it+1}. The price of bonds q_{it} decreases with the expected default probability, implying a higher interest rate for borrowing. In the special case with a zero default probability, debt price $q_{it} = \frac{1}{1+r_2}$ where r_2 is the prime borrowing rate. The prime borrowing rate r_2 exceeds the saving rate r_1 by assuming a per-dollar intermediation cost, c_I. I use $b_{it+1} > 0$ to represent borrowings and $b_{it+1} < 0$ to represent savings.

Given the assumption that firms cannot issue equity, dividend d_{it} is nonnegative by the end of each period:

$$
\begin{aligned}
d_{it} = \Pi_{it}(z_{it}, k_{it}, m_{it}) - (k_{it+1} - (1-\delta)k_{it}) - C(k_{it}, k_{it+1}) - \omega m_{it+1} - \\
b_{it} + q_{it}(z_{it}, b_{it+1}, k_{it+1}, m_{it+1}) b_{it+1} - c_o \geq 0 \qquad (14)
\end{aligned}
$$

where c_o is the operating cost.

Value of Continuation For simplicity, the rest of the model is in a recursive form and abstracts away the firm subscript i.

At the beginning of each period, a firm chooses to continue or to exit. Given the state variables (z, b, k, m) and the bond price schedule $q'(z, b', k', m')$, the value of continuation is

$$
\begin{aligned}
& V^c(z, b, k, m) = \max_{b', k', m'} \Pi(z, k, m) + (1-\delta)k - \omega m' - k' - C(k, k') - b + q'(z, b', k', m')b' \\
& - c_o + \beta E_{z'|z} V(z', b', k', m') \qquad (15) \\
& s.t. \quad \Pi(z, k, m) + (1-\delta)k - \omega m' - k' - C(k, k') - b + q'(z, b', k', m')b' - c_o \geq 0 \\
& q'b' \leq E_{z'|z} V^0(z', k') \qquad \text{(No Ponzi-Game)} \qquad (16)
\end{aligned}
$$

where β is the discounting factor and cannot exceeds $\frac{1}{1+r_2}$, because otherwise firms borrow indefinitely. If $\beta < \frac{1}{1+r_2}$, firms only borrow when the investment on capital and intermediate inputs gives a return greater than $\frac{1}{\beta} - 1$. To make sure that firms borrow whenever the return is greater than the prime borrowing rate, I set $\beta = \frac{1}{1+r_2}$.

The constraint (16) imposes another state-dependent upper limit on borrowings, $E_{z'|z}V^0(z', k')$, which is the expected value of firms when there are only capital adjustment costs without financial frictions. The rationale is that borrowings cannot exceed the net present value of future dividends the firm creates. This constraint stops firms from playing a Ponzi-game.

Exit and Default At the end of production in the current period, the net worth of a firm is the sum of cash $\Pi(z,k,m) - b\mathbb{1}(b \leq 0)$ and the undepreciated capital $(1-\delta)k$ net debt repayment $b\mathbb{1}(b > 0)$. Once a firm decides to exit, $(1-\gamma_2)$ fraction of cash $\Pi(z,k,m) - b\mathbb{1}(b < 0)$, and $(1-\gamma_1)$ fraction of capital $(1-\delta)k$ evaporate with an assumption of $\gamma_2 < \gamma_1$. In other words, exit is costly for the whole economy. Given the limited liability, the exit value for a firm is:

$$V^x(z,b,k,m) = \max\{\gamma_2\Pi(z,k,m) - b[\mathbb{1}(b \leq 0)\gamma_2 + \mathbb{1}(b > 0)] + \gamma_1(1-\delta)k, 0\} \tag{17}$$

An endogenous exit $\chi(z,b,k,m) = 1$ happens when $V^x(z,b,k,m) > V^c(z,b,k,m)$. Therefore, the value function $V(z,b,k,m)$ of a firm before exit is $\max\{V^x(z,b,k,m), V^c(z,b,k,m)\}$.

Default only happens when firms exit, while borrowings are rolled over when firms continue.[24] A firm may also exit without default, if it saves $b \leq 0$, or if the liquidation value of capital and cash $\gamma_2\Pi(z,k,m) + \gamma_1(1-\delta)k$ exceeds the debt repayment b. The only case when an exiting firm defaults is that the liquidation value is smaller than the debt repayment. Loss for lenders is then $b - \gamma_2\Pi(z,k,m) - \gamma_1(1-\delta)k$.

2.2 Entrants and Firm Size Distribution

In each period t, there are $\mu_{ent}M_t$ mass of entrants. Each entrant draws an initial permanent productivity $\bar{z}$ from a distribution $N(0,\sigma_z^2)$, and a transitory productivity μ_0 from another distribution $N(0,\sigma_\mu^2)$. The entrant also draws an initial wealth $b_0 < 0$ independently from a Pareto distribution with the density function $g(-b_0)$:

$$g(-b_0) = \begin{cases} \frac{\alpha a_{\min}^{\alpha}}{(-b_0)^{\alpha+1}} & if -b_0 \geq a_{\min}, \\ 0 & if -b_0 < a_{\min}. \end{cases} \tag{18}$$

[24] There are other studies that allow debt defaults when firms continue. See Bai et al. (2018), for instance.

where a_{min} is the minimum wealth.

Firms do not enter and produce right away. There exists a preparation period for entrants to build up capital stock and intermediate inputs out of scratch, according to their initial productivities $z_0 = \bar{z} + \mu_0$ and wealth draws b_0. In other words, firms have zero initial capital stocks and intermediate inputs, $k_0 = 0, m_0 = 0$. Their choices of borrowing or saving $b'_{ent}(z_0, -b_0, 0, 0)$, capital $k'_{ent}(z_0, -b_0, 0, 0)$, and intermediate inputs $m'_{ent}(z_0, -b_0, 0, 0)$ for the first production period are given by maximizing the value function $V_{ent}(z_0, b_0, 0, 0)$

$$V_{ent}(z_0, b_0, 0, 0) = \max_{b',k',m'} -\omega m' - k' - b_0 + q'(z, b', k', m')b' - c_o + \beta E_{z'|z_0} V(z', b', k', m') \tag{19}$$

$$s.t. -\omega m' - k' - b_0 + q'(z, b', k', m')b' - c_o \geq 0 \tag{20}$$

$$q'b' \leq E_{z'|z} V^0(z', k') \qquad \text{(No Ponzi-Game)} \tag{21}$$

where the transitory component of next period z' evolves in the same AR(1) process as incumbents in Equation (11), and V is the value function of incumbents.[25] Note that there is no adjustment costs with a zero initial capital stock.

2.3 Financial Intermediaries

There exists a continuum of risk-neutral competitive intermediaries that take deposits and lend. Given debt price functions $q'(z, b', k', m')$, the problem for a competitive lender is to choose a supply function $b'^s = b'^s(z, k', m'; q')$ to maximize its expected profit:

$$\begin{aligned} \max_{b'} (1 - E_{z'|z}\chi'(z', b', k', m'))b' + E_{z'|z}\{\chi'(z', b', k', m')(b' - \gamma_2 \Pi(z', k', m') \\ - \gamma_1(1-\delta)k')\} - (1 + r_1 + c_I)q'b' \end{aligned} \tag{22}$$

[25] Unlike studies such as Hopenhayn (1992), Cooley and Quadrini (2001) and Bento and Restuccia (2015), this paper does not model endogenous entries. In general equilibrium, output price channels affect the value of entry and pin down the equilibrium mass of entrants through equating value of entry to entry costs. Given the absence of price channels in a partial equilibrium framework, I take the equilibrium mass of entrants over distributions of productivity and wealth as given, which are later parametrized to match the data.

The first term here presents debt repayment b'^{s} with the probability of $1 - E_{z'|z}\chi'(z', b', k', m')$ that firms continue and debt is rolled over. The second term gives an expected loss when the borrower defaults.

To summarize, Section 2 presents a model composed of infinitely-lived firms and financial intermediaries. Firms produce and maximize net present values of dividends, given frictions on intermediate inputs and capital. Firms choose whether to exit and default every period. Financial intermediaries ex-ante set a break-even interest rate that reflects this default probability. The equilibrium in the loanable funds market, the entries and exits shape the industrial dynamics.

3 Quantitative Analysis

This section implements quantitative analysis. Section 3.1 describes how I parametrize the model to match key moments in the AIES data. Section 3.2 quantifies and compares measured misallocation in the model and in the data. Section 3.3 decomposes misallocation generated by each friction in the model. Section 3.4 discusses economic mechanisms through which intermediate input frictions cause misallocation, and the difference of measured misallocations in the current approach versus the conventional Hsieh and Klenow (2009)'s approach.

3.1 Parametrization

I first introduce the mapping between the model and the data, given that AIES is a selective sample and only covers the top 20% manufacturing firms in sales over 1998-2007. According to the First Economic Census 2004, the average nominal sales and capital stock of Chinese manufacturing firms are 6.64 and 2.84 million yuan, far below the averages of 39.63 and 16.47 million yuan in AIES in the same year. The left-truncation of sales also biases entries and exits in this dataset. Over a 5-year time window, more than 30% of the seeming entrants in AIES are incumbent firms. These firms have a non-negligible market share close to 15% in the year of entering AIES.[26]

Given these facts, I simulate firms from the model implied stationary distributions and obtain the top 20% subsample in sales, which I treat as the AIES model analog. I shall point out that in the model simulated sample, intermediate inputs usage $\tilde{m}$, not the pre-paid level m, corresponds to the observed firm-level intermediate inputs in AIES. Combined with entrants' wealth and productivity distributions, one could simulate a 5-year unbalanced panel of firms that looks like the AIES data.

In terms of parameters, I first parametrize capital adjustment costs with a fixed cost parameter $\xi = 0.039$ and a convex cost parameter $\theta = 0.049$ following Cooper and Haltiwanger (2006). Capital deprecation rate δ equals to 0.09. Firms' discount factor β is set to 0.94, which implies an average prime borrowing interest rate $r_2 = 0.06$ according to People's Bank of China (PBOC) annual reports over 1998-2007. Similarly, the saving interest rate r_1 equals to 0.03 to match the average deposit rate in PBOC reports.

The remaining parameters are calibrated. In the gross output production function, the labor and intermediate input shares, α_l and α_m, are set to the average wage bill and intermediate revenue shares in AIES, 0.05 and 0.70. Since the capital share is unobserved, I calibrate the return to scale parameter η to match the fact that 84.5% of total gross output is produced by the top 10% firms in the manufacturing sector, which are equivalently the top 50% firms in AIES. The rationale is that as η increases, gross output in the economy is more concentrated within the largest firms. This gives $\eta = 0.85$ and consequently $\alpha_k = 0.10$.

The population exit rate differs from the exit rate in AIES, and is determined by the operating cost c_o. I set c_o to match the population exit rate 8% during 2008-2012, according to a firm survival analysis report by the State Administration for Industry and Commerce of China. The annual growth rate in the manufacturing population during this period is approximately 9%, according to the economic censuses of 2004 and 2008. I choose the annual growth rate and the exit rate from time frames different from 1998-2007 because

[26] A firm is identified as an incumbent if the reported opening year is before the first year it appears in AIES. This statistic is computed as an average over two periods, 1998-2003 and 2002-2007. See Table A3-A4 in appendix for more details.

these are the most available estimates, to the best of my knowledge. The relative mass of entrants μ_{ent} is hence 17% to match this growth rate.The sales threshold y_c is 584.15, such that 20% of firms are above this level in the simulated gross output distribution.

Capital and cash recovery rates, γ_1 and γ_2, are crucial to determine how binding the borrowing constraint is. I calibrate γ_2 to match the standard deviation of the marginal revenue products of intermediate inputs. Jointly, γ_1 is calibrated to match the correlation between the debt level and the capital stock. The idea is that when γ_1 increases, borrowings using capital stock as a superior form of collateral increases, and hence increases the correlation. This gives $\gamma_1 = 0.50, \gamma_2 = 0.10$. These numbers are slightly higher than the average asset recovery rates of non-performing loans in the book *Capitalizing China* (Fan and Morck, 2013, p. 85), which reports recovery rates of 30% for capital, and 6.9% for cash for the four state-owned asset management companies.

The productivity process parameters are calibrated to match the productivity moments in the top 20% sample to those in AIES. I discretize the permanent productivity $\bar{z}_i$ into 5 grids, and the transitory productivity μ_{it} into 15 grids, using the Tauchen (1986)'s method. The persistence of transitory productivity ρ and its standard deviation are chosen to match the one-period persistence and the cross-sectional dispersion of productivities in the data. The mean and standard deviation of the permanent productivity distribution are jointly calibrated to match average and 5-year period persistence of firm-level productivities in the data.

For entrants, the productivity distribution of entrants is the same as that of incumbents. The shape parameter α and minimum wealth a_{min} of the initial wealth distribution determine the first-period output for entrants after entry. The fraction of intermediate inputs paid a period ahead ω impacts how fast a firm grows post entry, and therefore the relative market share over different ages. Thus, the three parameters, namely, α, a_{min} and ω are jointly determined to match the facts that 6.94% of newly-established firms younger than five years old have sales greater than y_c, that these firms in AIES are 65.56% of an average AIES firm in sales and that 37.09% of AIES entrants are older than five, over a 5-year period in the data.[27]

[27]The ratios of 37.09% and 65.56% are averages for two time periods, 1998-2003 and 2002-2007 (see Table

Table 4: Model Parametrization

Parametrized			Calibrated		
Parameter		Value	Parameter		Value
Discounting factor	β	0.94	Return to scale	η	0.85
Depreciation rate	δ	0.09	Labor share	α_l	0.05
Capital adjustment cost			Intermediate input share	α_m	0.70
Fixed cost	ξ	0.039	Fraction of intermediate inputs in advance	ω	40%
Convex cost	θ	0.049	Threshold sales	y_c	584.15
Interest rates			Operating cost	c_o	0.30
Saving rate	r_1	0.03	*Recovery rates*		
Prime borrowing rate	r_2	0.06	Capital	γ_1	0.50
			Cash	γ_2	0.10
			Transitory productivity		
			Persistence	ρ	0.80
			Standard deviation	σ_ϵ	0.18
			Permanent productivity		
			Mean	$\mu_{\bar{z}}$	0.90
			Standard deviation	$\sigma_{\bar{z}}$	0.40
			Initial wealth distribution of entrants		
			Mass of entrants	μ_{ent}	0.17
			Pareto shape	α	50.00
			Minimum wealth	a_{min}	8.00

Table 5: Targeted Moments

Moments	Data	Model
All firms		
Market share by firms of top 10% sales	84.50%	87.50%
Exit rate	8.00%	8.38%
Frac. of firms above threshold	20.00%	20.00%
Top 20%		
SD($MRPM$)	0.2720	0.2653
Corr(debt, capital)	0.7834	0.4764
Corr(productivity z, productivity lag 1 z_{-1})	0.6086	0.6620
Average productivity z	1.9770	1.9680
SD(productivity z)	0.4253	0.3625
5-year unbalanced & top 20%		
Corr(productivity z, productivity lag 5 z_{-5})	0.3898	0.1984
Fraction of newly established firms above threshold	6.94%	5.56%
Fraction of incumbents in end-year entrants	37.09%	56.03%
Relative size of newly established firms	65.56%	73.80%

Notes: Correlation between debt and capital is only for firms that borrow. In AIES, firm-level debt is measured by total liabilities. Over a 5-year period, newly-established firms are the ones with ages younger than five. End-year entrants are the ones that enter into the subsample with sales higher than the threshold, regardless of whether they are incumbents or newly-established firms.

Table 4 lists all calibrated parameters and their values, and Table 5 shows the differences of targeted moments in the model and in the data. The model overall well replicates the data in exit rates, the dispersion of marginal products in intermediate inputs, and the one-period productivity persistence. Nevertheless, there are several moments that the model can hardly fit, such as the correlation between debt and capital, the 5-year persis-

A4 in appendix). The ratio of 6.94% equals 66,221 firms younger than five years old in AIES in 2003, divided by the total number of newly-established firms over five years. This denominator is estimated as 953,388, assuming a 17% entry rate, an 8% exit rate, and that AIES also comprised the top 20% manufacturing firms in 1998.

tence of productivities, and the fraction of growing incumbents that enter into the top 20% subsample by the end of 5-year periods.

3.2 Misallocation: Model vs Data

This subsection computes and compares measured misallocations in the model simulated firm-level data and in AIES.

I first discuss the case of reallocating intermediate inputs alone. Recall the exercise in Table 1 of Section 1.2 that reallocates intermediate inputs alone to compute gross output and value-added gains, holding capital and labor fixed. The same exercise in the subsample of the top 20% firms in the model simulated data gives a gross output gain of 5.37% and a value-added gain of 13.97% in the left panel of Table 6. These are about the same magnitude of the intermediate input misallocation in AIES. Table 6 further suggests that there is perhaps a higher intermediate input misallocation in China's manufacturing sector than that in AIES. Potential gross output and value-added gains for all firms increase by 2.19% and 0.84% additionally on top of the gains in AIES.

The next question is to see how much the model accounts for measured gross output and value-added misallocation in the data. I define the gross output misallocation by the percentage of total gross output gain when capital, labor, and intermediate inputs are reallocated to equalize their marginal revenue products across firms, holding the total amount of input supplies constant. The value-added misallocation is defined as the percentage of total value-added gain in the above process. In AIES, the reallocation is done within 2-digit industries as in Section 1.2.

The right panel of Table 6 suggests that the model accounts well for the measured misallocation in the data. Gross output for an average 2-digit industry would increase by 21.64% and value-added by 89.86% if there were no dispersions of marginal revenue products for inputs in AIES. These numbers in the model simulated top 20% subsample are 21.84% and 58.87%. Thus, the model accounts for 100% of the gross output misallocation and 65% of the value-added misallocation in the data. The difference between these two percentages is due to the fact that I set 0.7 as the undistorted intermediate input

Table 6: Output Gains by Reallocation of intermediate inputs Alone, Simulated and Actual

	Reallocating intermediates		Reallocating intermediates, capital and labor	
	Gross output	Value-added	Gross output	Value-added
AIES	4.98%	20.61%	21.64%	89.86%
Top 20%, simulated	5.37%	13.97%	21.84%	58.87%
All, simulated	6.21%	16.16%	23.06%	62.17%

Notes: Numbers from AIES are averaged over 1998-2007.

revenue share, and the actual intermediate input share is smaller than 0.7 with frictions in the model. Yet in the data, the actual share in the data 0.74 and there are firms with distorted intermediate inputs revenue share both above and below 0.7.

Table 6 also suggests that quantifying the misallocation in AIES gives a downward bias on the amount of misallocation for China's manufacturing sector. For all simulated firms in the model, the gross output and value-added would increase more, by 23.06% and 62.17%, respectively. This result is sensible since firms below the cutoff sales are more financially constrained, as suggested by their lower capital stock and higher interest rates in Table A5. In fact, the standard deviation of $MRPM_i$ for all firms is 1.9819, much higher than the calibration target 0.2953 among the top 20% firms.

3.3 Decomposing Misallocation

There are four frictions in the benchmark model: borrowing constraints on capital and intermediate inputs, pre-pay on intermediate inputs, and capital adjustment costs. How much does each friction account for the measured misallocation in the calibrated model? More importantly, do intermediate input frictions help to account for more misallocation, in addition to that caused by capital frictions in a standard investment model? To answer these questions, I implement several counterfactual experiments that remove subsets of frictions.

The left panel of Table 7 illustrates what frictions are removed for each experiment. Experiment 1 removes the borrowing constraints on intermediate inputs. Experiment 3 removes the borrowing constraints on capital, and Experiment 4 removes capital adjustment costs. Since I embed borrowing constraints on intermediate inputs through pre-pay, it is infeasible to remove the pre-pay friction alone. Experiment 2 then removes the borrowing constraints and the pre-pay friction on intermediate inputs together. Experiment 5 removes the borrowing constraints and adjustment costs on capital. Experiment 6 removes all frictions, and only retains the one-period time-to-build friction on capital. Thus, comparing the amount of misallocation in each experiment to the benchmark model gives the magnitude of the misallocation caused by the removed friction(s).

In this partial equilibrium framework, the levels of output, intermediate inputs, capital, and labor are not directly comparable across experiments. Thus, I quantify gross output and value-added misallocations by computing the reallocation gains among simulated firms for each experiment, which are re-generated using calibrated parameters. The idea is to see how much the static misallocation there would be if firms hypothetically lived in the counterfactual economy.

Table 7 presents the misallocation results on its right panel. Note that across experiments, total intermediate inputs usage as a share of total gross output varies. The share is 70% in Experiment 2 and 6 when the intermediate input frictions are absent. However, when there are intermediate input frictions in the Benchmark model, in Experiment 3, 4, and 5, the share drops to distorted levels smaller than 70%.[28] Such a difference matters for the value-added misallocation since it equals to gross output misallocation divided by one net the actual intermediate input revenue share. Therefore, when I isolate misallocation caused by certain frictions, I discuss cases with the distorted α_m and the undistorted α_m. The former could be viewed as purely the static misallocation, while the latter adds

[28] One concern is how the lower and distorted intermediate inputs share in this model reconciles with China's high aggregate share of intermediate inputs. Admittedly, the model is not able to generate distortions $\tau_{is}^m < 0$ in Figure 1, i.e., when firms use too much intermediate inputs. The emphasis of one-sided distortion in the model hence lowers the economy-wide revenue share. This is not contradictory to the empirical argument that China adopts a technology that employs more intermediate inputs than other countries.

extra gains by increasing total intermediate inputs to an undistorted level.

I start discussions of misallocation generated by certain friction(s) among the top 20% firms. Comparison between Benchmark and Experiment 6 implies that one-period time-to-build capital friction combined with stochastic productivities drives the most gross output and value-added misallocations, about 60% and 74% in the Benchmark model, respectively. These numbers are unexpectedly high, given the rich specification of frictions in the model. However, this is generally consistent with Asker et al. (2014), which find that the dynamic nature of capital, rather than the level of adjustment costs, accounts most for the cross-country misallocation differences.[29]

If I compute the misallocation caused by each friction, Table 8 suggests that a similar level of importance for the borrowing constraint on intermediate inputs compared to that on capital. Differencing the Benchmark and Experiment 1 implies that an additional 5.12% potential gross output gain when borrowing constraints on intermediate inputs exist. The corresponding additional value-added gains with an undistorted α_m is 17.08%. These numbers are 4.89%, and 16.29% for borrowing constraints on capital. When I use the actual distorted α_m, value-added misallocation generated by the borrowing constraints on intermediate inputs drops to 5.84%, because intermediate inputs revenue share increases from 63% in the Benchmark model to 68% in Experiment 1. The value-added misallocation by borrowing constraints on capital using a distorted α_m does not drop much, because intermediate inputs are also distorted in Experiment 3.

When I group frictions by the input they affect, Table 8 suggests that frictions on each input generate about a similar magnitude of misallocation compared to borrowing constraints on that input. Table 8 also implies that the misallocation generated by financial frictions could be quantitatively doubled when intermediate inputs are also affected.

If I further focus on all simulated firms instead, the magnitudes of gross output and

[29]In a related paper David and Venkateswaran (2019), firms receive a noisy signal of productivity shocks for the next period, and are hence partially informed of the next period productivity in addition to the information of current productivity z_{it}. Firms make capital investment according to the signal. Because of this semi-static nature of capital, their estimate of misallocation induced by pre-determined capital is smaller than this paper.

value-added misallocations from four frictions are larger in row (1) of Table 8. Qualitatively and quantitatively, the importance of borrowing constraints on intermediate inputs and capital in accounting for the misallocation still holds among all simulated firms.

Table 7: Simulated Gains by Equalizing Marginal Products, Benchmark and Counterfactuals

	B.C. on M	Pre-pay on M	B.C. on K	Adj. cost on K	Gains, top 20% Gross output	Gains, top 20% Value-added	Gains, all Gross output	Gains, all Value-added
Data					21.64%	89.86%	-	-
Model								
Benchmark					21.84%	58.87%	23.06%	62.17%
Exp 1	×				16.72%	53.03%	17.06%	54.15%
Exp 2	×	×			16.47%	54.88%	16.47%	54.88%
Exp 3			×		16.95%	46.68%	18.45%	50.86%
Exp 4				×	20.83%	57.15%	22.73%	62.38%
Exp 5			×	×	17.31%	47.48%	18.48%	50.75%
Exp 6	×	×	×	×	13.11%	43.69%	13.25%	44.18%

Notes: B.C. on M means borrowing constraints on intermediate inputs. B.C. on K means borrowing constraints on capital. Pre-pay on M means pre-pay friction on intermediate inputs. Adj. costs on K means capital adjustment costs. Cross in each cell of the left panel specifies which friction is removed from the Benchmark model.

3.4 Discussions

This subsection discusses economic mechanisms in the benchmark model, and how I map the current approach to the conventional value-added approach of misallocation as in Hsieh and Klenow (2009).

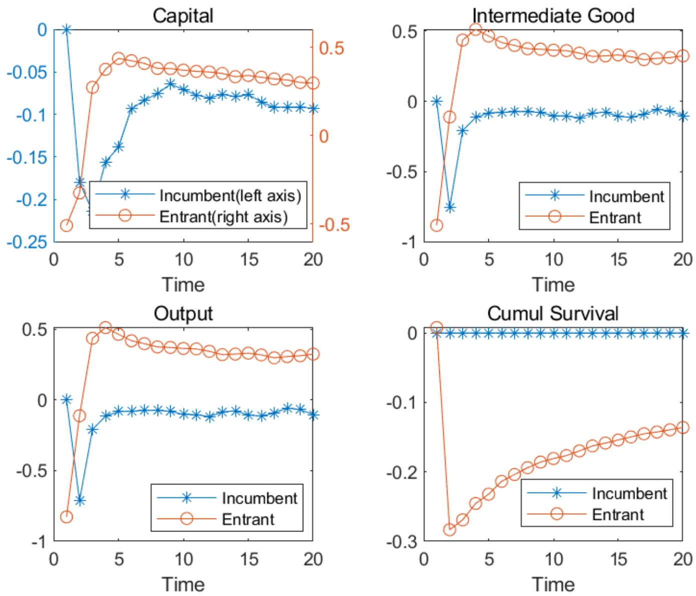
Role of Intermediate Input Frictions I discuss the role of intermediate input frictions for incumbents and entrants separately. The experiment is as follows. At $t = 1$, I first sample incumbents from the stationary distribution in Experiment 2 and entrants from the joint distribution of initial productivities and wealth.

Table 8: Contribution of Each Friction to Misallocation

	Gains, top 20%			Gains, all		
	Gross output	Value-added (Undistorted α_m)	Value-added (Distorted α_m)	Gross output	Value-added (Undistorted α_m)	Value-added (Distorted α_m)
All frictions (1)	8.73%	29.11%	15.18%	9.81%	32.70%	17.99%
Single friction						
B.C. on M (2)	5.12%	17.08%	5.84%	6.00%	20.01%	8.02%
B.C. on K (3)	4.89%	16.29%	12.19%	4.61%	15.37%	11.31%
Adj. Cost on K (4)	1.01%	3.38%	1.72%	0.33%	1.11%	-0.22%
By input						
Intermediates (5)	5.38%	17.92%	3.98%	6.60%	21.99%	7.28%
Capital (6)	4.53%	15.11%	11.39%	4.58%	15.27%	11.42%

Notes: B.C. on M means borrowing constraints on intermediate inputs. B.C. on K means borrowing constraints on capital. Adj. costs on K means capital adjustment costs. Undistorted α_m means that the aggregate intermediate inputs as a percentage of aggregate gross output is undistorted by frictions and equals to 70%. Distorted α_m means the actual aggregate intermediate inputs revenue share in the simulated data. The entry of value-added misallocation in the last column of Row (4) is negative because the actual intermediate inputs revenue share is higher in Experiment 4 than in Benchmark.

Figure 2: Firms' Response to One-period Intermediate Input Frictions

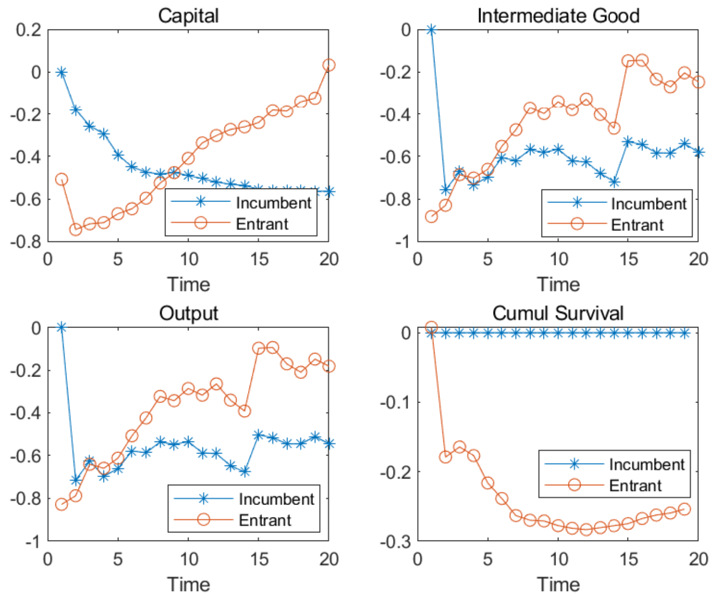


Notes: Values on Y-axis of the first three figures mean percentage gaps of capital, intermediate good and output, compared to firms in Experiment 2. Values on Y-axis of the last figure mean differences in cumulative survival rates compared to firms in Experiment 2.

In the next step, two versions of the experiment are implemented. In the first version, incumbents and entrants are unexpectedly imposed intermediate input frictions for one period at $t = 1$. Starting from $t = 2$, these frictions are then removed unexpectedly. This version aims to highlight the negative impact of intermediate input frictions on firms. In the second version, both incumbents and entrants are unexpectedly imposed intermediate input frictions starting from $t = 1$ permanently. This version shows how the per-period intermediate input frictions could reinforce intertemporally, and slow down firms' growth.

Figure 2 plots firms average capital, intermediate input usage, output and cumulative survival after one-period shock of frictions for 20 periods. I compare these levels to the same set of firms that live in Experiment 2. For incumbents, extra financing for intermediate inputs induces a lower capital investment. Thus, at $t = 2$, capital, intermediate inputs and output are around 18%, 75% and 71% lower, respectively, than firms in Experiment 2. Firms do not exit more because of the shock of frictions. When the frictions are

Figure 3: Firms' Response of Permanent Intermediate Input Frictions



Notes: Values on Y-axis of the first three figures mean percentage gaps of capital, intermediate good and output, compared to firms in Experiment 2. Values on Y-axis of the last figure mean differences in cumulative survival rates compared to firms in Experiment 2.

removed from $t = 2$, firms choose the static optimal level of intermediate inputs to maximize per-period profit starting from $t = 3$. Firms hence recover gradually in capital due to adjustment costs and financial constraints. Starting from $t = 3$, capital, intermediate inputs and output are steadily 10% lower than the levels in Experiment 2. Overall, the negative effect of one-period intermediate good frictions is persistent.

Entrants are slightly different. Note that I impose intermediate input frictions after they draw initial productivities and wealth when they plan for the first period production. At $t = 1$ when the entrants start to produce, their capital is about 50% lower, while their intermediate inputs and output are more than 80% lower, compared to the same set of entrants in Experiment 2. Firms borrow more and the average borrowing increases from 128 to 376 by almost 3 times. As a result, exits increase and the cumulative survival rate is 28% lower at $t = 2$, compared to Experiment 2. Such a strong cleansing effect makes the remaining firms selectively productive. Hence, average levels of intermediate inputs and output exceed those in Experiment 2 for the later periods.

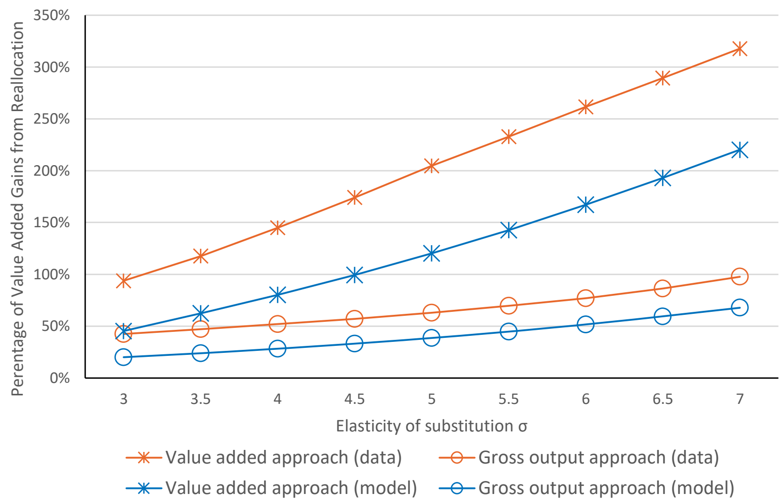
In the second version of the experiment, firms are imposed intermediate input frictions, starting from $t = 1$ permanently. Figure 3 plots average capital, intermediate input usage, output and cumulative survival rates for 20 periods in this version. Compared to the earlier case, incumbent firms are even smaller, and converge to an average size that is about 60% smaller than the size of firms Experiment 2 in terms of capital, intermediate input usage and output. The average level of capital stock for entrants grows at a much slower rate, and converges to the level of entrants in Experiment 2 at $t = 20$. Because of the intermediate input frictions, levels of intermediate inputs and output for entrants are steadily below those in Experiment 2. Similar to Figure 2, exit rates are higher for entrants but not for incumbents. These results suggest that the per-period intermediate input frictions reinforce their negative effects intertemporally and induce a greater misallocation in the benchmark model.

Comparison with the Value-added Approach Note that the value-added misallocation is defined as the percentage of total value-added gain when three inputs are reallocated. In their seminal paper, Hsieh and Klenow (2009) use a value-added production function, and quantify the potential value-added gain if dispersions of marginal revenue products of capital and labor are eliminated. A natural question is that how does the value-added misallocation under the gross output approach, compared to their numbers given the same firm-level dataset?

To answer the question, I replicate their results using their elasticity of substitution $\sigma = 3$, cost shares of capital and labor 0.5, and the same trimming procedure. Value-added misallocation over 1998-2007 under this approach is 93.79%, i.e., GDP would increase by 93.79% if the marginal revenue products of capital and labor were equalized. This number is close to theirs.

Since I use $\sigma = 6.67$ in this paper, I then vary σ from 3 to 7 with the cost shares of capital and labor unchanged. Figure 4 shows that the amount of misallocation under the value-added approach increases quickly in the data and is very sensitive to the level of σ. When $\sigma = 7$, value-added misallocation triples to a level of 317.71%. The monotonic increase is due to an increasing loss caused by input misallocation when products across

Figure 4: value-added Misallocation by Two Approaches, Model (Benchmark) and Data



Notes: Log quantity and revenue productivities are trimmed by the top and bottom 1% in the data for both approaches. In model simulated data, no trimmings are applied under the assumption that the model should replicate variations in trimmed data. Capital and labor shares are 0.5 in the model and data. Alternative capital and labor shares give similar trends.

firms become more substitutable.

However, if I plot the measure of value-added misallocation under the gross output approach, the levels decline substantially.[30] Value-added misallocation is now 43% when $\sigma = 3$ and increases to 98% when $\sigma = 7$ in the data. Figure 4 then plots the measures of value-added misallocation in the benchmark model using the two approaches with different σs.[31] One can see that measures of value-added misallocation using the two approaches in the model closely track those in AIES .

Why misallocation measures are substantially higher under the value-added approach? Note that the value-added productivities $TFPQ_{is}^{v}$ and $TFPR_{is}^{v}$ could be written as functions of the gross output productivities $TFPQ_{is}$, intermediate input distortion τ_{is}^{m}, i.e.,

[30] A similar result is found in a recent paper by Hang et al. (2019) which put more emphasis on the input-output linkage across sectors.

[31] For model simulated data, I set the capital share as $\tilde{\alpha}_k$ divided by one net $\tilde{\alpha}_m$ and the labor share similarly for the value-added approach. Results are similar if I set them as 0.5.

$logMRPM_{is}$, and capital distortion τ^k_{is}, i.e., $logMRPK_{is}$

$$log(TFPQ^v_{is}) = log(TFPQ_{is}) - \frac{\alpha^s_m}{\sigma - 1}\tau^m_{is} + \frac{\tilde{\alpha}^s_k \tilde{\alpha}^s_m}{1 - \tilde{\alpha}^s_m}\tau^k_{is} + C_1 \quad (23)$$

$$log(TFPR^v_{is}) = \frac{\tilde{\alpha}^s_k}{1 - \tilde{\alpha}^s_m}\tau^k_{is} + \alpha^s_m \tau^m_{is} + C_2 \quad (24)$$

where C_1 and C_2 are constants. In the appendix, I show that if $logTFPQ_{is}$, τ^k_{is} and τ^m_{is} follow a joint normal distribution, the measured value-added gain is

$$\frac{0.5}{1 - \alpha^s_m}\{[\tilde{\alpha}^{s2}_k(\sigma - 1) + \tilde{\alpha}^s_k]VAR(\tau^k) + [\tilde{\alpha}^{s2}_m(\sigma - 1) + \tilde{\alpha}^s_m]VAR(\tau^m) + 2\tilde{\alpha}^s_k\tilde{\alpha}^s_m(\sigma - 1)COV(\tau^k, \tau^m)\} \quad (25)$$

under the gross output approach, and is

$$\frac{0.5}{1 - \alpha^s_m}\{[\frac{\tilde{\alpha}^{s2}_k}{1 - \tilde{\alpha}^s_m}(\sigma - 1) + \tilde{\alpha}^s_k]VAR[(1 - \alpha^s_m + \alpha^s_m\tau^m)\tau^k]\} \quad (26)$$

under the value-added approach. In the extreme case when $\tau^m_{is} = 0$ for all firms, Equation (26) suggests that returns to capital in the value-added specification show a smaller dispersion than $VAR(\tau^k)$ with the multiplier $1 - \alpha^s_m$. Hence, the misallocation measure in Equation (26) would be smaller, but not larger than that in Equation (25).

When intermediate input distortions exist, however, the distortion adds additional variations to capital returns in the value-added specification. The increased $VAR[(1 - \alpha^s_m + \alpha^s_m\tau^m)\tau^k]$ combined with a larger coefficient of capital $\frac{\tilde{\alpha}^s_k}{(1-\tilde{\alpha}^s_m)}$ in the value-added production function pushes up the misallocation measure in (26). While intermediate input distortions also increase the measure of misallocation in the second and third term of equation (25), this effect could be smaller when $VAR(\tau^k)$ is sufficiently larger than $VAR(\tau^m)$. In both model and data, for instance, $VAR(\tau^k)$ are more than 50 times as large as $VAR(\tau^m)$. Because of the distortion τ^m, the dispersion of returns to capital in the value-added specification, i.e., $VAR[(1 - \alpha^s_m + \alpha^s_m\tau^m)\tau^k]$, is about the same magnitude of $VAR(\tau^k)$, but not much smaller as in the case $\tau^m = 0$.

The result above is not likely due to the constant supply of intermediate inputs within each industry under the gross output approach. In the pseudo general equilibrium experiment of Section 1, I show that when I increase intermediate inputs supply by λ-fold, the value-added gain in fact decreases by diminishing returns. Further, it is not clear that under the value-added approach, whether the supply of intermediate inputs increases or decreases after reallocations because of its perfect substitutability with the value-added bundles.

These results suggest the potential of intermediate input frictions in accounting for the sizable misallocation found in the previous literature. Using the value-added output measure, existing work finds dynamic models of firms hard to generate a large output loss at a magnitude close to that in the data (e.g., Midrigan and Xu, 2014; Moll, 2014). However, if one looks at the data through the lens of firms as gross output producers and allows the reallocation of intermediates, the misallocation measure declines substantially and becomes more robust to the change in elasticities of substitutions. Moreover, combined with results in Section 3.3, the measured misallocation via the new lens becomes closer to the output loss generated through dynamic models of firms with the help of explicitly modeled intermediate input frictions.

To summarize, Section 3 first calibrates the model to the AIES data, and finds that the model well replicates measured misallocations in AIES when one reallocates intermediate inputs alone, and along with capital and labor. Second, Section 3 finds that the misallocation generated by borrowing constraints doubles when both intermediate inputs and capital are affected. Third, Section 3 discusses how the misallocation measures using the gross output approach compare to those using the conventional value-added approach.

4 Conclusion

Most of the existing literature on misallocation study distortions in firm-level inputs of capital and labor. This paper introduces the intermediate input frictions and builds a quantitative model to evaluate their relative importance in accounting for misallocation in China's AIES data.

This paper contributes to the literature in three folds. It first empirically quantifies the intermediate input misallocation that are comparable to the well-understood capital misallocation. Second, it shows the negative consequence of financial frictions on the aggregate output could be doubled in a dynamic firm investment model, when financial frictions also constrain the intermediate inputs. Lastly, using the lens of firms as gross output producers, one would obtain the potential value-added gain in a firm-level dataset smaller than the conventional value-added approach. Combined with the second contribution, it implies that the gap could be narrowed between the sizable misallocation quantified in firm-level datasets and the moderate level a dynamic model generates in the previous literature.

There are several directions for future work. First, the idea that firms may be constrained in intermediate inputs could be applied to other developing economies and recession stages of developed economies (Kehrig, 2015). Second, this paper takes the partial equilibrium framework to understand the first-order effect of intermediate input frictions. One could close the economy using the input-output structure across industries. How would this inter-sector linkage amplify output losses, following the idea of Jones (2011)? What about the relative magnitudes of a direct output loss as in this paper, and an indirect loss by the linkage? These questions are aligned with the research agenda in understanding how distortions and shocks affect the GDP in an inter-connected economy (Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi, 2012; Bartelme and Gorodnichenko, 2015; Osotimehin and Popov, 2020).

References

Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2012), "The Network Origins of Aggregate Fluctuations." *Econometrica*, 80, 1977–2016.

Ackerberg, Daniel A., Kevin Caves, and Garth Frazer (2015), "Identification Properties of Recent Production Function Estimators." *Econometrica*, 3, 2411–2451.

Antràs, Pol, Davin Chor, Thibault Fally, and Russell Hillberry (2012), "Measuring the upstreamness of production and trade flows." *American Economic Review*, 102, 412–16.

Antras, Pol and C Fritz Foley (2015), "Poultry in motion: a study of international trade finance practices." *Journal of Political Economy*, 123, 853–901.

Asker, John, Allan Collard-Wexler, and Jan De Loecker (2014), "Dynamic Inputs and Resource (Mis)Allocation." *Journal of Political Economy*, 122(5), 1013–1063.

Bai, Yan, Dan Lu, and Xu Tian (2018), "Do financial frictions explain chinese firms saving and misallocation?" *NBER Working Paper No. 24436*.

Baqaee, David Rezza and Emmanuel Farhi (2020), "Productivity and misallocation in general equilibrium." *The Quarterly Journal of Economics*, 135, 105–163.

Bartelme, Dominick and Yuriy Gorodnichenko (2015), "Linkages and Economic Development." *NBER Working Paper No.21251*.

Bartlesman, Eric, John Haltiwanger, and Stefano Scarpetta (2013), "Cross-Country Differences in Productivity: The Role of Allocation and Selection." *American Economic Review*, 103(1), 305–334.

Bento, Pedro and Diego Restuccia (2015), "Misallocation, Establishment Size, and Productivity." *2015 Meeting Papers. No. 1391, Society for Economic Dynamics*.

Bils, Mark, Peter J. Klenow, and Cian Ruane (2017), "Misallocation or Mismeasurement?" *Working Paper*.

Boehm, Johannes and Ezra Oberfield (2018), "Misallocation in the Market for Inputs: Enforcement and the Organization of Production." *NBER Working Paper No. 24937*.

Brandt, Loren, Gueorgui Kambourov, and Kjetil Storesletten (2018), "Barriers to Entry and Regional Economic Growth in China." *Manuscript, University of Toronto*.

Brandt, Loren, Trevor Tombe, and Xiaodong Zhu (2013), "Factor Market Distortions across Time, Space and Sectors in China." *Review of Economic Dynamics*, 16, 39 – 58.

Brandt, Loren, Johannes Van Biesebroeck, and Zhang (2014), "Challenges of Working with the Chinese NBS Firm-level Data." *China Economic Review*, 30, 339–352.

Brandt, Loren, Johannes Van Biesebroeck, and Yifan Zhang (2012), "Creative Accounting or Creative Destruction? Firm-level Productivity Growth in Chinese Manufacturing." *Journal of Development Economics*, 97, 339–351.

Braun, Matias (2003), "Financial Contractibility and Asset Hardness." *University of California-Los Angeles mimeo*.

Broda, Christian and David E Weinstein (2010), "Product creation and destruction: Evidence and price implications." *American Economic Review*, 100, 691–723.

Buera, Francisco and Yongseko Shin (2013), "Financial Frictions and the Persistence of History: A Quantitative Exploration." *Journal of Political Economy*, 121(2), 221–272.

Buera, Francisco J., Joseph P. Kaboski, and Yongseok Shin (2015), "Entrepreneurship and Financial Frictions: A Macrodevelopment Perspective." *Annual Review of Economics*, 7, 409–436.

Caselli, Francesco and Nicola Gennaioli (2013), "Dynastic Management." *Economic Inquiry*, 51, 971–996.

Cooley, Thomas F. and Vincenzo Quadrini (2001), "Financial Markets and Firm Dynamics." *American Economic Review*, 91(5), 1286 – 1310.

Cooper, Russell W. and John C. Haltiwanger (2006), "On the Nature of Captial Adjustment Costs." *Review of Economic Studies*, 73, 611–633.

Cuñat, Vicente (2007), "Trade credit: suppliers as debt collectors and insurance providers." *The Review of Financial Studies*, 20, 491–527.

David, Joel M., Hugo A. Hopenhayn, and Venky Venkateswaran (2016), "Information, Misallocation, and Aggregate Productivity." *Quarterly Journal of Economics*, 131, 943–1005.

David, Joel M. and Venky Venkateswaran (2019), "Capital Misallocation: Frictions or Distortions?" *NBER Working Paper No. 23129*.

De Loecker, Jan and Frederic Warzynski (2012), "Markups and Firm-Level Export Status." *American Economic Review*, 102(6), 2437–2471.

Deloof, Marc (2003), "Does working capital management affect profitability of belgian firms?" *Journal of business finance & Accounting*, 30, 573–588.

Fally, Thibault (2012), "Production staging: measurement and facts." *Boulder, Colorado, University of Colorado Boulder, May*, 155–168.

Fan, Joseph P.H. and Randall Morck (2013), *Capitalising China*. The University of Chicago Press.

Fazzari, Steven M. and Bruce C. Petersen (1993), "Working Capital and Fixed Investment: New Evidence on Financing Constraints." *The RAND Journal of Economics*, 24, 328–342.

Foster, Lucia, John C. Haltiwanger, and Chad Syverson (2008), "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" *American Economic Review*, 98(1), 394–425.

Gandhi, Amit, Salvador Navarro, and David Rivers (2017), "How Heterogeneous is Productivity? A Comparison of Gross Output and Value Added." *Journal of Political Economy, Forthcoming*.

Gopinath, Gita, Şebnem Kalemli-Özcan, Loukas Karabarbounis, and Carolina Villegas-Sanchez (2017), "Capital Allocation and Productivity in South Europe." *The Quarterly Journal of Economics*, 132(4), 1915–1967.

Haltiwanger, John, Robert Kulick, and Chad Syverson (2018), "Misallocation Measures: The Distortion that Ate the Residual." *NBER Working Paper No. 24199*.

Hang, Jing, Pravin Krishna, and Heiwei Tang (2019), "Production Networks, Trade and Misallocation." *Working paper*.

Hopenhayn, Hugo A. (1992), "Entry, Exit, and firm Dynamics in Long Run Equilibrium." *Econometrica*, 60(5), 1127–1150.

Hopenhayn, Hugo A. (2014), "Firms, Misallocation, and Aggregate Productivity: A Review." *Annual Review of Economics*, 6, 735–770.

Hsieh, Chang-Tai and Peter J. Klenow (2009), "Misallocation and Manufacturing TFP in China and India." *The Quarterly Journal of Economics*, 124 (4), 1403–1448.

Jones, Charles I. (2011), "Intermediate Goods and Weak Links in the Theory of Economic Development." *American Economic Journal: Macroeconomics*, 3(2), 1–28.

Jose, Manuel L., Carol Lancaster, and Jerry L. Stevens (1996), "Corporate Returns and Cash Conversion Cycles." *Journal of Economics and Finance*, 20(1), 33–46.

Kehrig, Matthias (2015), "The Cyclical Nature of the Productivity Distribution." *Working Paper*.

Krusell, Per and Anthony A Smith (1998), "Income and wealth heterogeneity in the macroeconomy." *Journal of Political Economy*, 106, 867–896.

Manova, Kalina (2013), "Credit Constraints, Hetereogenous Firms, and International Trade." *Review of Economic Studies*, 80, 711–744.

Mendoza, Enrique G. and Vivian Z. Yue (2012), "A General Equilibrium Model of Sovereign Default and Business Cycles." *The Quarterly Journal of Economics*, 127, 889–946.

Miao, Jianjun and Pengfei Wang (2014), "Sectoral Bubbles, Misallocation, and Endogenous growth." *Journal of Mathematical Economics*, 53, 153–163.

Midrigan, Virgiliu and Daniel Yi Xu (2014), "Finance and Misallocation: Evidence from Plant-Level Data." *American Economic Review*, 104(2), 422–458.

Moll, Benjamin (2014), "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation." *American Economic Review*, 104(10), 3186–3221.

Oberfield, Ezra and Devesh Raval (2014), "Micro data and macro technology." Technical report, National Bureau of Economic Research.

Osotimehin, Sophie and Latchezar Popov (2020), "Misallocation and intersectoral linkages." In *Working Paper*.

Peters, Michael (2013), "Heterogeneous Mark-ups, Growth and Endogenous Misallocation." *Working Paper, The London School of Economics and Political Science*.

Petersen, Mitchell A. and Raghuram G. Rajan (1997), "Trade Credit: Theories and Evidence." *Review of Financial Studies*, 10, 661–691.

Pratap, Sangeeta and Carlos Urrutia (2012), "Financial Frictions and Total Factor Productivity: Accounting for the Real Effects of Financial Crises ." *Review of Economic Dynamics*, 15, 336 – 358.

Raddatz, Claudio E (2003), "Liquidity needs and vulnerability to financial underdevelopment." *Journal of Financial Economics*, 80, 677–722.

Rajan, Raghuram and Luigi Zingales (1998), "Financial development and growth." *American Economic Review*, 88, 559–586.

Ramey, Valerie A (1989), "Inventories as factors of production and economic fluctuations." *The American Economic Review*, 338–354.

Rauch, James E. (1999), "Networks versus markets in international trade." *Journal of International Economics*, 48(1), 7–35.

Restuccia, Diego and Richard Rogerson (2013), "Misallocation and Productivity." *Review of Economic Dynamics*, 16, 1–10.

Richards, Verlyn D and Eugene J Laughlin (1980), "A cash conversion cycle approach to liquidity analysis." *Financial management*, 32–38.

Song, Zheng, Kjetil Storesletten, and Fabrizio Zilibotti (2011), "Growing Like China." *American Economic Review*, 101, 196 – 233.

Tauchen, George (1986), "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions." *Economics Letters*, 20, 177 – 181.

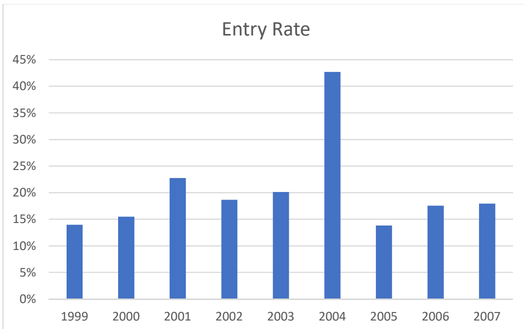
Tombe, Trevor and Xiaodong Zhu (2019), "Trade, Migration, and Productivity: A Quantitative Analysis of China." *American Economic Review*, 109, 1843 – 1872.

Tong, Hui and Shang-Jin Wei (2011), "The composition matters: capital inflows and liquidity crunch during a global economic crisis." *The Review of Financial Studies*, 24, 2023–2052.

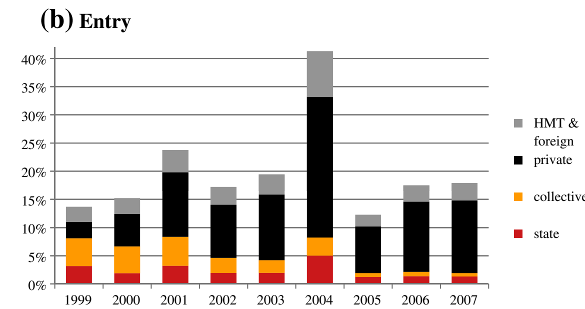
Uras, Burak R. and Ping Wang (2018), "Production Flexibility, Misallocation and Total Factor Productivity." *NBER Working Paper No. 23970*.

Figure A1: Entry and Exit Rates in AIES Data, Compared to Brandt et al. (2012)

(a) Entry: This Paper

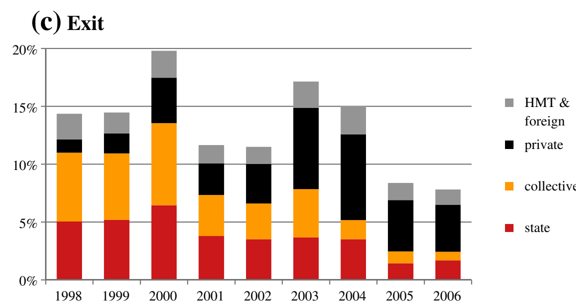


(b) Entry: Brandt et al. (2012)



(c) Exit: This Paper

(d) Exit: Brandt et al. (2012)



Notes: Entrants (exits) include (1) firms enter into (exit from) production as in the model; (2) firms grow above (decline below) 5 million threshold sales.

Appendix

Matching Firms across Years AIES is cross-sectional and requires matching of firms over time to understand the dynamics. I match firms across years based on codes in Brandt et al. (2012) with some modifications. Firms are matched sequentially according to their registration IDs, names, a string that combines names of legal representatives, industry and area codes, a string that combines phone numbers, industry and area codes, and lastly a string that combines opening year, geographic code, industry, township/streets/villages and product names. Entry and exit rates in the merged data are plotted in Figure A1 in comparison to Brandt et al. (2012). One can see that matching results are close. Note that entries and exits in the AIES data are not equivalent to those in the model. I also use the same perpetual inventory method from their paper to construct the firm-level real capital stock over time.

One-Input Reallocation This appendix introduces how Section 1.2 calculates gross output and value-added gains by reallocating one input at a time. To simplify the notation, I use A_{is} to represent $exp(z_{is})$ in the paper. Note the industry-level $T\bar{F}PR_s$

$$T\bar{F}PR_s = \frac{P_s Y_s}{K_s^{\tilde{\alpha}_k^s} L_s^{\tilde{\alpha}_l^s} M_s^{\tilde{\alpha}_m^s}} = \frac{\sum_{i=1}^{N_s} P_{is} Y_{is}}{K_s^{\tilde{\alpha}_k^s} L_s^{\tilde{\alpha}_l^s} M_s^{\tilde{\alpha}_m^s}} \tag{27}$$

and hence the industry-level productivity

$$\begin{aligned}
TFP_s =& \frac{Y_s}{K_s^{\tilde{\alpha}_k^s} L_s^{\tilde{\alpha}_l^s} M_s^{\tilde{\alpha}_m^s}} = T\bar{F}PR_s / P_s \\
=& T\bar{F}PR_s \Big(\sum_{i=1}^{N_s} \big(\frac{A_{is}}{TFPR_{is}}\big)^{\sigma-1}\Big)^{\frac{1}{\sigma-1}} \\
=& \Big(\sum_{i=1}^{N_s} \big(A_{is} \frac{T\bar{F}PR_s}{TFPR_{is}}\big)^{\sigma-1}\Big)^{\frac{1}{\sigma-1}} \\
=& \Big(\sum_{i=1}^{N_s} \big(A_{is} \frac{M\bar{R}PK_s^{\tilde{\alpha}_k^s} M\bar{R}PM_s^{\tilde{\alpha}_m^s} M\bar{R}PL_s^{\tilde{\alpha}_l^s}}{MRPK_{is}^{\tilde{\alpha}_k^s} MRPM_{is}^{\tilde{\alpha}_m^s} MRPL_{is}^{\tilde{\alpha}_l^s}}\big)^{\sigma-1}\Big)^{\frac{1}{\sigma-1}}
\end{aligned} \tag{28}$$

Note that $A_{is} = \frac{R_{is}^{\frac{\sigma}{\sigma-1}}}{K_{is}^{\tilde{\alpha}_k^s} L_{is}^{\tilde{\alpha}_l^s} M_{is}^{\tilde{\alpha}_m^s}} (P_s^{\sigma} Y_s)^{-\frac{1}{\sigma-1}}$, where R_{is} is firm-level gross output revenue in the data. According to Equation (28), industry TFP_s in the data could be computed with a multiplier $(P_s^{\sigma} Y_s)^{-\frac{1}{\sigma-1}}$.

If only intermediate inputs are reallocated to equalize its marginal revenue products within industries, $M\bar{R}PM_s = MRPM_{is}$ holds after the reallocation. The reallocation, however, changes the firm-level gross output revenue

$$R'_{is} = (P_{is} Y_{is})' = A_{is} K_{is}^{\tilde{\alpha}_k^s} L_{is}^{\tilde{\alpha}_l^s} M_{is}^{'\tilde{\alpha}_m^s} (Y'_s P_s^{'\sigma})^{\frac{1}{\sigma}} \tag{29}$$

where R'_{is} or $(P_{is} Y_{is})'$, M'_{is}, Y'_s, and P'_s denote revenue, intermediate inputs, industry-level output quantity, industry-level price after the reallocation. Firm-level and industry-level marginal revenue products of capital and labor also change accordingly. The co-movement of industry- and firm-level marginal products helps to cancel out $(Y'_s P_s^{'\sigma})^{\frac{1}{\sigma}}$ in equation (28) after reallocation. The new industry TFP_s is

$$TFP'_s = (\sum_{i=1}^{N_s} (A_{is} \frac{M\bar{R}PK_s^{'\tilde{\alpha}_k^s} M\bar{R}PL_s^{'\tilde{\alpha}_l^s}}{MRPK_{is}^{'\tilde{\alpha}_k^s} MRPL_{is}^{'\tilde{\alpha}_l^s}})^{\sigma-1})^{\frac{1}{\sigma-1}} \tag{30}$$

where $MRPX'$ is the new marginal product, $x = K, L$. Thus, gross output gain for industry s is

$$TFP'_s/TFP_s - 1 \tag{31}$$

which cancels out $(P_s^\sigma Y_s)^{-\frac{1}{\sigma-1}}$ in both A_{is}s in the numerator and in the denominator. The economy-wide gross output gain is an industry gross output weighted gains

$$\sum_{s=1}^{S} \theta_s TFP'_s/TFP_s - 1 \tag{32}$$

Gross output gains from reallocating capital or labor are computed similarly.

To compute the value-added gain, note that the industry-level value-added VA_s

$$VA_s = Y_s - M_s \tag{33}$$

The economy-wide value-added gain after reallocating intermediate inputs is

$$\frac{\prod_{s=1}^S (Y'_s)^{\theta_s} - \prod_{s=1}^S (Y_s)^{\theta_s}}{\prod_{s=1}^S (Y_s)^{\theta_s} - \sum_{s=1}^S M_s} \tag{34}$$

The numerator is the gross output gain, and the denominator is the value-added before the reallocation. Note

$$\begin{aligned}\frac{\prod_{s=1}^S Y_s^{\theta_s}}{\sum_{s=1}^S M_s} &= \frac{P\prod_{s=1}^S Y_s^{\theta_s}}{P\sum_{s=1}^S M_s} \\ &= \frac{\sum_{s=1}^S P_s Y_s}{P\sum_{s=1}^S M_s} \\ &= \frac{1}{\alpha_m}\end{aligned} \tag{35}$$

The second equation holds because of the zero-profit condition for the Cobb-Douglas aggregator, and α_m is the economy-wide intermediate inputs revenue share. Therefore, the value-added gain is

$$\sum_{i=1}^{S} \frac{\theta_s}{1-\alpha_m}(TFP'_s/TFP_s - 1) \tag{36}$$

Misallocation in FHS(2008) Industries This appendix quantifies the input misallocation in industries that feature a homogeneous output studied in Foster et al. (2008). I exclude white pan bread, roasted coffee beans, and oak flooring since these products are not consumed much by Chinese households over 1998-2007. Since the finest 4-digit CIC industries are still coarser than these products, I keep firms that report the product as what they produce and what they *only* produce using the information from three variables, Main Product 1, Main Product 2 and Main Product 3 in AIES. For example, I keep firms that report their Main Product 1 variable as the Chinese translation of Corrugated & Solid Fiber Boxes, i.e., *Zhi Xiang*, and their Main Product 2 and 3 missing. The product of gasoline is dropped since numbers of firms that uniquely produce gasoline are fewer than 10 for most years.

Financially Vulnerable Measures Table A2 lists asset tangibility and external finance dependence measures for 28 2-digit industries. Figure A2 plots coefficients of variations in $MRPM_{ist}$ against these financially vulnerable measures in the year of 2004.

Table A1: Input Misallocation in Foster et al. (2008) Industries

	Gross output gains			Value-added gains		
	Intermediates	Capital	Labor	Intermediates	Capital	Labor
Corrugated & Solid fiber Boxes	4.51%	4.57%	1.89%	17.81%	16.78%	6.81%
Carbon Black	3.13%	3.15%	1.08%	11.77%	11.63%	3.89%
Ready Mixed Concrete	4.31%	3.77%	2.20%	15.83%	14.03%	8.35%
Plywood	1.21%	21.67%	2.19%	3.35%	55.05%	6.21%
Sugar	6.23%	1.77%	0.47%	33.99%	9.66%	2.83%

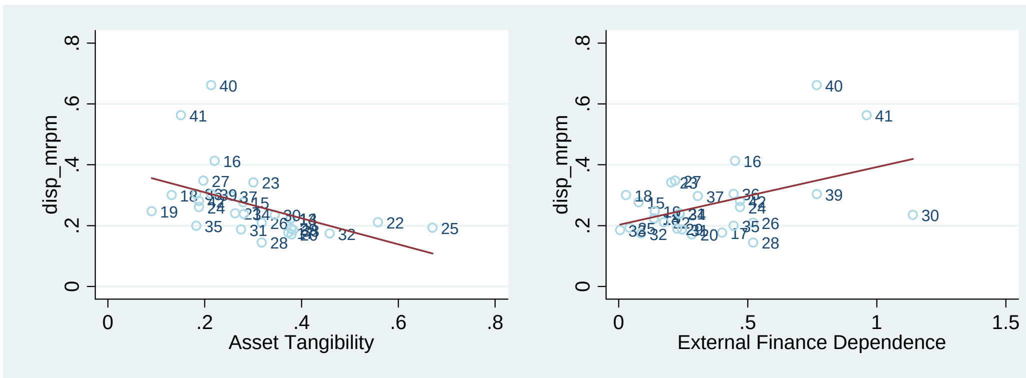
Note: Misallocation measures are gains obtained when reallocating one input at a time, using the same method as in Table 1. Numbers are averaged from 1998 to 2007. The year of 2002 for carbon black and sugar is dropped for intermediate inputs, since gains are much higher than the rest of years. For the same reason, the year of 1998 for ready mixed concrete is dropped.

Table A2: Asset Tangibility and External Finance Dependence Measures in 2-digit CIC Industries

2-digit CIC Code	Industry Name	Asset Tangiblities	External Finance Dependence
13	Primary Food Processing	0.3777	0.1368
14	Manufacture of food	0.3777	0.1368
15	Manufacture of beverage	0.2794	0.0772
16	Manufacture of tobacco	0.2208	-0.4512
17	Manufacture of textile	0.3730	0.4005
18	Garments and products	0.1317	0.0286
19	Leather, furs and down products	0.0906	-0.1400
20	Timber processing	0.3796	0.2840
21	Manufacture of furniture	0.2630	0.2357
22	Paper making and products	0.5579	0.1756
23	Printing and publishing	0.3007	0.2038
25	Petroleum refineries	0.6708	0.0420
26	Raw chemical materials	0.4116	0.2050
27	Medical and pharmaceutical products	0.1973	0.2187
28	Chemical fiber	0.1973	0.2187
29	Rubber products	0.3790	0.2265
30	Plastic products	0.3448	1.1401
31	Nonmetal mineral products	0.4200	0.0620
32	Ferrous metals	0.4581	0.0871
33	Nonferrous metals	0.3832	0.0055
34	Metal products	0.2812	0.2371
35	Ordinary machinery	0.1825	0.4453
37	Transport equipment	0.2548	0.3069
39	Electric equipment and machinery	0.2133	0.7675
40	Electronic and telecom equipment	0.2133	0.7675
41	Professional and scientific equipment	0.1511	0.9610
42	Other manufacturing	0.1882	0.4702

Notes: Numbers are from Table I in Braun (2003).

Figure A2: Industry-level Dispersion of Marginal Revenue Products of Intermediates, Financial Vulnerabilities and Operating Cycles, Year 2004



Note: $disp\_mrpm_s$ for industry s is the standard deviation of $MRPM_{is}$ standardized by its industry-level mean, after trimming the top and bottom 1%. Measures of asset tangibility and external finance dependence are from Braun (2003) and Rajan and Zingales (1998) that use the Compustat data. Year 2004 is an example but results are similar for other years.

Table A3: Output, Capital, Employment and Firm Age in AIES and Census, 2004

	AIES		Census	
	Mean	S.D.	Mean	S.D.
Sales	39,626.59	74,143.66	6,643.77	17,272.79
Capital	16,465.49	37666.58	2,847.01	8,108.27
Employment	223	740	64	343
Age	8.22	10.14	6.48	8.11
Number	252,095		1,255,707	

Notes: Sales and capital are in 1000 yuan, current price.

Table A4: Entry in AIES Dataset over 5 Years

	China	
	1998-2003	2002-2007
Number of Firms, End Year		
AIES Incumbents	39.90%	30.27%
AIES Entrants (Age >5)	23.32%	24.67%
AIES Entrants (Age ≤ 5)	36.77%	45.06%
Market Share, End Year		
AIES Incumbents	60.45%	57.34%
AIES Entrants (Age >5)	14.66%	14.07%
AIES Entrants (Age ≤ 5)	24.89%	28.59%

AIES and Census 2004 The AIES data 1998-2007 has a threshold sales of 5 million yuan. Table A3 describes the differences between AIES and the Economic Census 2004. One can see that AIES firms are the largest 20% of all manufacturing firms in the Census. Average sales, capital, and employment in AIES are almost six times as large as those in the Census data. Firms in the Census are also younger on average. Nevertheless, AIES produces about 90% of manufacturing output and employs 70% of workers.

Inventory Model This appendix presents an alternative framework that models the inventory stock of intermediate inputs, and hence the purchases and consumptions of intermediates as positive and negative changes of this inventory.

Let the state variable m now be the intermediate input inventory. Suppose that in the current period, the firm makes $\Delta m'$ intermediates investment and uses $\tilde{m}$ for productions. The intermediate inventory for the next period is hence $m' = m - \tilde{m} + \Delta m'$ when there is no inventory holding cost or no wearing out.

Because the firm still needs to pay $(1-\omega)\Delta m$ in this period, purchase of intermediate inputs in the previous period Δm becomes the additional state variable. Given the whole state vector $(z, b, k, m, \Delta m)$, value of continuation is

$$
\begin{aligned}
V^c(z, b, k, m, \Delta m) = max_{b',k',\Delta m'} max_{\tilde{m} \leq m}\{py(\tilde{m}) - (1-\omega)\Delta m + m - \tilde{m}\} + (1-\delta)k - \omega \Delta m' - k' - C(k,k \\
& q'b' - c_o + \beta E_{z'|z} V(z', b', k', m', \Delta m') \\
& s.t. \quad max_{\tilde{m} \leq m}\{py(\tilde{m}) - (1-\omega)\Delta m + m - \tilde{m}\} + (1-\delta)k - \omega \Delta m' - k' - C(k, k') - b + q'b' - c_o \\
& m' = m - \tilde{m} + \Delta m' \\
& q'b' \leq E_{z'|z} V^0(z', k') \qquad \text{(No Ponzi-Game)}
\end{aligned}
\tag{37}
$$

Although the original model is not differentiable near the hyperplane where $V^x = V^c$, I add a stochastic shock $\Theta \in [\underline{\Theta}, \bar{\Theta}]$ to the value of exit, V^x, to make the problem differentiable here for an ease of exposition, i.e.,

$$
\begin{aligned}
V^x(z, b, k, m, \Delta m) = max\{\gamma_2 max_{\tilde{m} \leq m}\{py(\tilde{m}) - (1-\omega)\Delta m + m - \tilde{m}\} - b[\mathbb{1}(b \leq 0)\gamma_2 + \mathbb{1}(b > 0)] \\
+ \gamma_1(1-\delta)k, \Theta\}
\end{aligned}
\tag{38}
$$

One can show that first-order conditions for the choice of $\Delta m'$ in this problem are the same as those for the next period intermediate inputs m' in the original model. Furthermore, the value of exit also keeps unchanged when $m = \Delta m$, and increases when $m > \Delta m$ compared to the original model. For the value of continuation, again dividend in this period keeps unchanged when $m = \Delta m$, and increases when $m > \Delta m$ that would alleviate borrowing constraints and the pre-pay friction. This channel increases the value of continuation because of the additional inventory dimension firms could choose. Since $m > \Delta m$ happens more often when productivity z_{-1} in the previous period is low, the extent on how much the inventory channel would alleviate borrowing constraints is likely to be small, especially if I further add an inventory holding cost. Hence, I use the original model for keeping the number of state variables small to avoid the curse of dimensionality in computation.

Computations The model is a partial equilibrium one since (i) there are no market clearings for final output and intermediate inputs, neither for capital and labor; (ii) the prime interest rate is exogenously given as in a small open economy. The partial equilibrium framework makes the computation relatively inexpensive, compared to a general equilibrium models à la Krusell and Smith (1998).

The computation has three parts: (i) value function iterations; (ii) finding a stationary firm size distribution (FSD); (iii) simulation based on FSD.

Value function iterations are computed in the following step:

1. Compute the value of continuation $\bar{V}^c$ as the maximum value when there are no financial frictions;

2. Let the initial guess $V_0^c = \bar{V}^c$. Note that one needs to guess from a value function higher than the true one everywhere because of the endogenous borrowing constraint and the existence of debt upper limit when the break-even interest rate might approach infinite;

3. Given V_0^c and V^x as in the paper, compute optimal b', k', m' and ensure that constraints of non-negative dividends and no Ponzi-Game are satisfied;

4. Update the value function and repeat 3 until the convergence criterion $||V_{n+1}^c - V_n^c|| < \epsilon$ is satisfied.

I then iterate to obtain a stationary firm size distribution on grids of debt, capital, intermediate inputs, and productivities. This distribution is then used to sample incumbent firms. Combined with a mass of μ_{ent} entrants sampled from initial debt and productivity distributions, one could obtain an AIES-alike unbalanced panel of firms that the reallocation exercises are based on.

I use gfortran 7.5 and OpenMPI 4.0.1 for parallel computations that employ 80 workers at a time. The number of grids is 5 for permanent productivities, 15 for transitory productivities, 16 for debt/saving, 16 for capital, 40 for intermediate inputs. The whole running time is 10 hours for each parameterization.

Constrained Firms in the Model and in the Data Firms with an $MRPM_i$ greater than 1 in the model either have unexpected high productivities because of the pre-pay, or are financially constrained and under-invest in intermediate inputs. In the latter case, if the model well captures the data, one would expect that firms with $MRPM_i > 1$ in the model and their counterparts in the data are similar. Table A5 thus compares the levels of capital

Table A5: Average Productivity, Capital and Interest Rates, Constrained and Unconstrained

	AIES		Top 20%, Simulated		All, Simulated	
	Constrained	Unconstrained	Constrained	Unconstrained	Constrained	Unconstrained
Log TFPQ	2.14	1.89	1.89	1.58	1.15	1.21
Capital Stock	23,153.83	24,373.31	1720.03	2715.37	308.05	1187.64
Interest Rate	6.43%	4.02%	6.20%	6.19%	267.66%	8.94%

Notes: Constrained firms have $MRPM_i > 1$, and unconstrained ones have $MRPM_i \leq 1$. Statistics in AIES are calculated first for each year, and are then averaged over 1998-2007. In AIES, capital stock are in constant 1998 thousand yuan, and interest rates are calculated by interest expenses divided by total liabilities.

stock, productivity and interest rates between firms with $MPRM_{it} > 1$ (constrained) and those with $MRPM_{it} \leq 1$ (unconstrained), both in the model and in the data. To remove the impact of state-owned firms, I restrict to the non-state-owned sector in AIES. And to meaningfully discuss financial constraints, I restrict my model-simulated data to the subsample of firms with debt, excluding firms that save.

The model resembles AIES in several aspects. Constrained firms, on average, have higher productivities, lower capital stocks, and pay higher interest rates both in the data and in the simulated top 20% subsample. For $TFPQ$, constrained firms are 25% and 31% more productive than unconstrained one in the data and in the model, respectively. The model delivers this phenomenon of size-dependent distortions because of a higher demand in intermediate inputs for more productive yet financially constrained firms. Firms whose sales are below the cutoff are less productive, and might be constrained for a different reason: their inability to generate cash flows from productions. Hence, for the entire simulated sample, constrained firms are actually 6% less productive than unconstrained ones. But this does not eliminate the size-dependent distortions. Correlation between marginal revenue products of intermediate inputs and productivity in the entire sample is still positive and equals to 0.02, although much smaller than 0.58 in the top 20% subsample.

For capital stock, constrained firms are 5% and 36% smaller than unconstrained firms in the data and in the top 20% subsample, respectively.[32] Accordingly, constrained firms pay interests rates that are 2.41% and 0.01% higher in the data and in the model. The phenomenon of a lower capital stock and higher interest rates for constrained firms is more pronounced in the entire simulated sample.

Value-added Misallocation: Two Approaches This appendix shows that misallocation measures, i.e., value-added gains through reallocations, differ under value-added and gross output approaches when the elasticity of substitution σ is finite.

Let me denote A^v_{is} and $TFPR^v_{is}$ as firm-level $TFPQ$ and $TFPR$ under the value-added approach. The corresponding industry-level $TFPQ$ and $TFPR$ are $\bar{A}^v_s$ and $T\bar{F}PR^v_s$. Note

$$log(A^v_{is}) = log(\frac{P_{is}Y_{is} - PM_{is}}{P_{is}}) - \frac{\tilde{\alpha}^s_k}{1-\tilde{\alpha}^s_m}log(K_{is}) - \frac{\tilde{\alpha}^s_l}{1-\tilde{\alpha}^s_m}log(L_{is}) \tag{39}$$

$$log(TFPR^v_{is}) = log(P_{is}Y_{is} - PM_{is}) - \frac{\tilde{\alpha}^s_k}{1-\tilde{\alpha}^s_m}log(K_{is}) - \frac{\tilde{\alpha}^s_l}{1-\tilde{\alpha}^s_m}log(L_{is}) \tag{40}$$

and $\alpha^s_k + \alpha^s_l + \alpha^s_m = 1 - \frac{1}{\sigma}$, $\tilde{\alpha}^s_k + \tilde{\alpha}^s_l + \tilde{\alpha}^s_m = 1$.

Using the property of Cobb-Douglas functions

$$\begin{aligned} log(P_{is}Y_{is} - PM_{is}) &= log(P_{is}Y_{is} - (1-\tau_{is})PM^*_{is}) \\ &= log(P_{is}Y_{is} - (1-\tau_{is})\alpha^s_m P_{is}Y_{is}) \\ &\approx log(P_{is}Y_{is}) - (1-\tau_{is})\alpha^s_m \end{aligned} \tag{41}$$

where M^*_{is} is the static optimal level of intermediate inputs, of which the revenue share shall equal to α^s_m in a Cobb-Douglas production function. τ^m_{is}, or $logMRPM_{is}$ equivalently, measures the idiosyncratic distance of actual intermediate inputs usage from the optimal level. Plugging this equation back to (39) and (40), the followings hold

$$log(A^v_{is}) = \frac{1}{1-\tilde{\alpha}^s_m}log(A_{is}) - \frac{\tilde{\alpha}^s_m}{(1-\tilde{\alpha}^s_m)(\sigma-1)}logM_{is} + \{\alpha^s_m - \frac{\tilde{\alpha}^s_m}{1-\tilde{\alpha}^s_m}\frac{\sigma}{\sigma-1}\}\tau^m_{is} + C'_1 \tag{42}$$

[32] In AIES, there is a time trend that constrained firms become increasingly smaller relative to the unconstrained ones in terms of capital stock. For instance, constrained firms are 9% larger in 1998. But in 2007, constrained firms are 23% smaller compared to the unconstrained ones, presumably because more smaller firms enter as economic reforms deepen.

$$log(TFPR_{is}^{v}) = \frac{1}{1-\tilde{\alpha}_m^s} log(TFPR_{is}) + \{\alpha_m^s - \frac{\tilde{\alpha}_m^s}{1-\tilde{\alpha}_m^s}\}\tau_{is}^m + C_2' \tag{43}$$

where C_1' and C_2' are constants in terms of α_m^s, σ, aggregate price P and aggregate output Y. The economic intuition of the second term in Equation (42) is that the monopolistic firms charge markups over the marginal cost of intermediate inputs when they produce gross output, which is absent when firms are modeled as value-added producers that charge markups only over the marginal costs of capital and labor. Analogously, the industry-level $T\bar{F}PR_s^v$ is

$$log(T\bar{F}PR_s^v) = \frac{1}{1-\tilde{\alpha}_m^s} log(T\bar{F}PR_s) + \{\alpha_m^s - \frac{\tilde{\alpha}_m^s}{1-\tilde{\alpha}_m^s}\}\bar{\tau}_s^m + C_2' \tag{44}$$

where $\bar{\tau}_s^m$ is the industry average distortion on intermediate inputs.

Note that misallocation, i.e., the potential value-added gain through reallocations, is quantified by

$$log(TFP_{s,eff}^v) - log(TFP_s^v) = log\{\sum_{i=1}^{N_s}(A_{is}^v)^{\sigma-1}\}^{\frac{1}{\sigma-1}} - log\{(\sum_{i=1}^{N_s}(A_{is}^v \frac{T\bar{F}PR_s^v}{TFPR_{is}^v})^{\sigma-1})^{\frac{1}{\sigma-1}}\} \tag{45}$$

under the value-added approach, and by

$$\frac{1}{1-\alpha_m^s}\{log(TFP_s) - log(TFP_{s,eff})\} = \frac{1}{1-\alpha_m^s}\{log\{\sum_{i=1}^{N_s} A_{is}^{\sigma-1}\}^{\frac{1}{\sigma-1}} - log\{(\sum_{i=1}^{N_s}(A_{is}\frac{T\bar{F}PR_s}{TFPR_{is}})^{\sigma-1})^{\frac{1}{\sigma-1}}\}\} \tag{46}$$

under the gross output approach. Hence, equations (45) and (46) are equal when $\sigma = \infty$, and both measures of misallocations approach infinity since only the most productive firms shall produce all goods at a constant return to scale after reallocations.

In a more realistic case when $\sigma < \infty$, one can show that the value-added $TFPQ$ and $TFPR$ can be expressed by the model primitives

$$log(A_{is}^v) = log(A_{is}) - \frac{\alpha_m^s}{\sigma-1}\tau_{is}^m + \frac{\tilde{\alpha}_k^s\tilde{\alpha}_m^s}{1-\tilde{\alpha}_m^s}\tau_{is}^k + C_1 \tag{47}$$

$$log(TFPR_{is}^v) = \frac{\tilde{\alpha}_k^s}{1-\tilde{\alpha}_m^s}\tau_{is}^k + \alpha_m^s\tau_{is}^m + C_2 \tag{48}$$

when C_1 and C_2 are constants, and τ_{is}^k is the capital wedge $logMRPK_{is}$. Note how distortions enter into the value-added quantity productivity. The following results are useful in deriving these equations

$$\begin{aligned}
(1-\tau_{is}^k)^{1-\alpha_l-\alpha_m}(1-\tau_{is}^m)^{\alpha_m}\frac{\sigma-1}{\sigma}A_{is}^{\frac{\sigma-1}{\sigma}}K_{is}^{-\frac{1}{\sigma}} &\propto r \\
\Rightarrow K_{is} &\propto (1-\tau_{is}^m)^{\alpha_m\sigma}(1-\tau_{is}^k)^{(1-\alpha_l-\alpha_m)\sigma}A_{is}^{\sigma-1} \\
\Rightarrow M_{is} &\propto K_{is}\frac{1-\tau_{is}^m}{1-\tau_{is}^k} \\
&\propto (1-\tau_{is}^m)^{\alpha_m\sigma+1}(1-\tau_{is}^k)^{(1-\alpha_l-\alpha_m)\sigma-1}A_{is}^{\sigma-1}
\end{aligned} \tag{49}$$

Now I use the model primitives to derive the analytical expressions of misallocation measures under both approaches. Note the industry $TFPR$

$$\begin{aligned}
T\bar{F}PR_s &= (M\bar{R}PK_s)^{\tilde{\alpha}_k^s}(M\bar{R}PM_s)^{\tilde{\alpha}_m^s} \\
&= [\sum_{i=1}^{N_s}\frac{P_{is}Y_{is}}{P_sY_s}(1-\tau_{is}^k)]^{-\tilde{\alpha}_k^s}[\sum_{i=1}^{N_s}\frac{P_{is}Y_{is}}{P_sY_s}(1-\tau_{is}^m)]^{-\tilde{\alpha}_m^s}
\end{aligned} \tag{50}$$

where $M\bar{R}PK_s$ is defined as $\tilde{\alpha}_k^s P_sY_s/K_s$, and $M\bar{R}PM_s$ as $\tilde{\alpha}_m^s P_sY_s/M_s$. Given K_{is}, M_{is} and the monopolistic competition market structure, one can show that

$$\frac{P_{is}Y_{is}}{P_sY_s} \propto \frac{A_{is}^{\sigma-1}(1-\tau_{is}^k)^{\tilde{\alpha}_k^s(\sigma-1)}(1-\tau_{is}^m)^{\tilde{\alpha}_m^s(\sigma-1)}}{\sum_{i=1}^{N_s}A_{is}^{\sigma-1}(1-\tau_{is}^k)^{\tilde{\alpha}_k^s(\sigma-1)}(1-\tau_{is}^m)^{\tilde{\alpha}_m^s(\sigma-1)}} \tag{51}$$

Assume that $a_{is} = log(A_{is})$, τ_{is}^m and τ_{is}^k follow a multivariate normal distribution with a variance-covariance matrix $\sum$. I allow covariances between a_{is} and τs, and the covariance between τs be nonzero. The industry average $TFPR$ can be expressed by

$$\begin{aligned}
T\bar{F}PR_s = exp\{\sigma-1\}exp\{(\tilde{\alpha}_k^s\mu_{\tau^k}+\tilde{\alpha}_m^s\mu_{\tau^m}) - (\tilde{\alpha}_k^{s2}(\sigma-1)+0.5\tilde{\alpha}_k^s)VAR(\tau^k) - (\tilde{\alpha}_m^{s2}(\sigma-1)+0.5\tilde{\alpha}_m^s)VAR(\\
-(\sigma-1)\tilde{\alpha}_k^s COV(a,\tau^k) - (\sigma-1)\tilde{\alpha}_m^s COV(a,\tau^m) - 2\tilde{\alpha}_k^s\tilde{\alpha}_m^s(\sigma-1)COV(\tau^k,\tau^m)\}
\end{aligned} \tag{52}$$

by applying moment generating functions for multivariate normal distributions. Hence, the potential value-added gain under the gross output approach is

$$\frac{1}{1-\alpha_m^s}\{0.5(\tilde{\alpha}_k^{s2}(\sigma-1)+\tilde{\alpha}_k^s)VAR(\tau^k)+0.5(\tilde{\alpha}_m^{s2}(\sigma-1)+\tilde{\alpha}_m^s)VAR(\tau^m)+\tilde{\alpha}_k^s\tilde{\alpha}_m^s(\sigma-1)COV(\tau^k,\tau^m)\} \tag{53}$$

and the covariance terms between a and τs are canceled out. Similarly, the potential value-added gain under the value-added approach is

$$0.5\{[\frac{\tilde{\alpha}_k^{s2}}{(1-\tilde{\alpha}_m^s)^2}(\sigma-1)+\frac{\tilde{\alpha}_k^s}{(1-\tilde{\alpha}_m^s)}]VAR[(1-\alpha_m^s+\alpha_m^s\tau^m)\tau^k]\} \tag{54}$$

where $\frac{\tilde{\alpha}_k^s}{(1-\tilde{\alpha}_m^s)}$ is the cost share of capital in a value-added production function, and $(1-\alpha_m^s+\alpha_m^s\tau^m)\tau^k$ is the firm-level value-added marginal revenue product $logMRPK^v$. It can be seen that intermediate inputs distortion adds another dimension of variation to returns to capital in a value-added specification. When the dispersion of τ^m increases, it is likely that equation (54) is going to increase faster with the first coefficient $\frac{\tilde{\alpha}_k^{s2}}{(1-\tilde{\alpha}_m^s)^2}$. However, it is hard to tell the relative magnitude in algebra and hence this paper turns to the data for the answer.