# Multi-Product Firms and Misallocation

Wenya Wang [*], Ei Yang[†]

School of Finance, Shanghai University of Finance and Economics

*June 21, 2021, First Draft: January, 2019*

**Abstract**

This paper studies how distortions alter the decisions of firms in producing multiple products (MP). Using the Annual Survey of Industrial Firm data in China, we find (i) multi-product firms are fewer and smaller than in the U.S.; (ii) the MP probability of a firm is negatively associated with the level of distortion. We build discrete product choices into a heterogeneous firm model à la Melitz (2003). A tax-like distortion increases the productivity cutoff of an MP firm, and vice versa for a subsidy-like distortion. The model is calibrated to firm-level data of China and the U.S. When distortions are removed, we find a comparable magnitude of welfare gains from the product extensive margin and from firm entry and exit. Both gains are sizable by the general equilibrium forces interacted with the granularity of firm size distributions.

**JEL Codes:** E44, L11, O11

[*]Email: wenyaw111@gmail.com.

[†]Email: yang.ei@mail.shufe.edu.cn. We thank comments and suggestions from seminar participants at AEA 2021, Asian Econometric Society Meeting 2019, The 2nd China Economic Development Workshop (Fudan) 2020, and Zhejiang University and Tsinghua University. All errors are on us.

# 1 Introduction

Across firms, the efficiency of input allocation is vital for a country's TFP level, and the pervasive distortions in developing economies decrease this efficiency (e.g., Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Midrigan and Xu, 2014). Within firms, the international trade literature emphasizes the product margin of export behaviors for firms with different productivity levels (e.g., Eckel and Neary, 2010; Bernard, Redding, and Schott, 2011; Mayer, Melitz, and Ottaviano, 2014; Manova and Yu, 2017). For the aggregate economy, according to Bernard, Redding, and Schott (2010) and Broda and Weistein (2010), the reallocation of resources across products could be empirically as important as the entry and exit margin of firms in developed economies. However, for developing economies, the empirical regularity on how firm-level product scopes are affected by distortions, and whether this product margin matters quantitatively for efficiency are not well understood.[1]

This paper fills up the gap and studies misallocation in a multi-product framework. Our empirical analysis utilizes the product information in China's Annual Survey of Industrial Firms (ASIF) firm-level data 1998-2007 from the National Bureau of Statistics (NBS) and defines products as 6-digit Chinese Industrial Classification Codes (5-digit SIC equivalent). Example products are copper wire (CIC 341203; SIC 33571) and passenger cars (CIC 370502; SIC 37111).[2]

We first find that multi-product firms are fewer and smaller than their counterparts in the U.S. Specifically, 27% of Chinese manufacturing firms are multi-product and they produce 44% manufacturing output in the ASIF data. When more small-

[1]There are two exceptions. Goldberg, Khandelwal, Pavcnik, and Topalova (2010) study multi-product firms in India. However, their dataset covers mainly listed firms, not the whole manufacturing sector. The other study, Jaef (2018), quantifies how distortions lower welfare via the multi-product channel in a quantitative macroeconomic model. The model, however, is not closely mapped to firm-level datasets in developing economies.

[2]For a full list, see the National Catalog of Product Classifications for Statistical Purposes from the NBS.

and medium-sized firms are included in the Economic Census 2004 data, these numbers drop to 17% and 42%. As a comparison, multi-product firms consist 39% of the U.S. manufacturing population, producing 87% of the manufacturing shipments, according to Bernard et al. (2010). An average multi-product (MP) firm is thus 10.46 times of an average single-product (SP) firm in the U.S. and at most 3.49 times in China. This finding of smaller MP firms is consistent with the observation of missing large firms in developing economies (Hsieh and Olken, 2014; Poschke, 2018; Bento and Restuccia, 2020).

Second, we find cross-sectional evidence on a negative relationship between the level of distortions and the probability of a firm producing multiple products. The distortion is indirectly proxied by the log TFPR, following Hsieh and Klenow (2009). We classify firms into six discrete types according to whether they are SP or MP and whether they are subsidized (i.e., a negative distortion), taxed below the median level, or taxed above the median level (i.e., a positive distortion). In the data, SP firms that are taxed high and low are 10% and 7% more productive than MP firms that are subsidized. This result contradicts theories that predict more products produced by more productive firms without distortions (e.g., Klette and Kortum, 2004; Bernard et al., 2010; Nocke and Yeaple, 2014) and suggests a distorted product margin. Furthermore, our Probit regressions in the ASIF data suggest that a one standard deviation increase in the distortion decreases the MP probability of a firm by 1.18 percentage points.

The distorted product margin is further confirmed when we follow a direct approach (Restuccia and Rogerson, 2013) and pick several specific distortions. In a difference-in-difference exercise, we exploit the natural experiment of 2008 corporate income tax reform that lowered the baseline rate from 33% to 25% and kept the rate unchanged for policy-favored programs (e.g., West Development Program). We set firms outside these programs before 2008 as the treatment group and find that the MP probability of the treated, relative to the controlled, increased by 1.2 percentage points after 2008. In the other exercise, we use the self-reported measures of distortions in the World Bank Enterprise Survey (WBES) data 2012 and check their relations with a firm's MP probability. We find that when firms report moderate, major or very severe obstacles in finance, transportation, custom, and trade regulations, the MP probability decreases by 5.65 to 9.94 percentage points compared to firms with no such obstacles.

The above results show the distorted product margin in the cross-sectional data. To understand the quantitative role of the product margin in general equilibrium, we build discrete multi-product choices into a canonical model of Melitz (2003) with firm entry and exit. In the model, the representative household supplies unskilled and skilled labor and derives a CES utility from a continuum of products produced within and across firms.[3] Firms are heterogeneous in productivities and product-level demand shocks for a maximum of two products.[4] Both productivities and demand shocks are fixed once drawn upon entry. Firms produce quantity and appeal (quality or taste) for each product. The quantity production function is linear in unskilled labor, while the appeal is produced by a decreasing return to scale function of skilled labor. The product appeal, multiplied by the quantity consumed, enters into the utility function of the household.

Firms pay an entry cost to enter, a firm-level and a product-level operating costs to produce. The profitability determines the strategy of a firm in producing zero, one, or two products. When firms produce two products, a span of control parameter governs the relative output of the second product. This setting reflects

[3] We assume the number of products is infinite, and the elasticities of substitution are the same between products within firms and across firms. This assumption makes each product line of a firm infinitely small and rules out the oligopolistic behavior of firms as emphasized in Eckel and Neary (2010) and Minniti and Turino (2013).

[4] We equate 2-product to multiple-product because of its simplicity and empirically at least two-thirds of the firm-level output for MP firms are produced by the top two competent products (Bernard et al., 2010).

the idea that producing extra products pulls firms away from their core competence (Mayer et al., 2014) and the degree of pulling away may differ across countries by the quality of management practices (Bloom, Manova, Van Reenen, Sun, and Yu, 2020). To capture the distortions in the data, firms additionally draw a time-invariant and firm-level output distortion that equals a positive level (i.e., taxed) when their productivity exceeds a threshold and a negative level (i.e., subsidized) otherwise, in a similar spirit to Guner, Ventura, and Xu (2008).

We then calibrate the distortion-free economy to match the U.S. economy and pin down parameters of the span of control and distortions to match the fraction and output share of MP firms, the top 5% output share, and the fraction of firms subsidized in the Chinese ASIF data. Compared to the hypothetical economy of a distortion-free China, a tax-like distortion increases the cutoffs of firms producing two products, and a subsidy-like distortion decreases the cutoffs. The distorted cutoffs are consistent with our findings in the cross-sectional data. When distortions are removed, the fraction of MP firms increases from 27.00% to 27.63% and their output share from 44.20% to 51.21%. The rest of the difference between distortion-free China and the U.S. is attributed to the span of control parameter. Specifically, if the parameter increases from 0.8 (China) to 1 (U.S.), the fraction of MP firms increases to 39.00% and their output share to 86.00%.

Although the distorted cutoffs contribute less to the fewer-and-smaller fact, they generate a sizable misallocation. To understand this channel, we implement three counterfactual experiments to decompose the welfare loss from misallocation into three margins: output intensive margin, product extensive margin, and firm extensive margin. Let the welfare level of a distortion-free China be 100%. Our distorted benchmark model generates a welfare level of 52.30%. In the first experiment (*PF-FF experiment*), we remove distortions but keep the mass of firms, product decisions, and the firm size distribution as fixed in the benchmark model. The welfare level increases to 60.71%. This gain of 8.41 percentage points is defined as *output intensive* *margin*, conceptually close to the static misallocation measure of Hsieh and Klenow (2009).

In the second experiment (*FF experiment*), we keep the mass and distribution of firms fixed as in the benchmark model but let firms re-optimize their product decisions. The welfare level increases to 95.24%, suggesting a *product extensive margin* of welfare loss by 34.53 (95.24-60.71) percentage points. Finally, in the third experiment (*PF experiment*), we let firms freely enter while keeping their product scope choices fixed as in the benchmark model. The difference in welfare levels between the PF experiment and the PF-FF experiment suggests a *firm extensive margin* of welfare loss by 36.57 (97.28-60.71) percentage points. The similar magnitude of misallocation by the product and firm extensive margins echoes the empirical finding in Bernard et al. (2010), which emphasizes the equal importance of product add-and-drop and firm entry-and-exit in the U.S. manufacturing production.

Welfare levels of 95.24% in the FF experiment and 97.28% in the PF experiment suggest a lack of complementarity between the two extensive margins. One may expect a further significant increase in welfare if more productive firms produce more products *and* if there are more of them. This lack of complementarity is driven by the general equilibrium forces combined with the granularity of the firm size distribution, which is an important feature of the firm-level data and is captured by our calibration.

The mechanism works as follows. In our model, the household values the product appeal, and firms monopolistically compete through the price-to-appeal ratio, not the price itself. High productivity firms seize market share by offering cheaper products with better appeals. Hence, they are much larger than in a model without endogenous appeal productions. When either the product or the firm margin is relaxed, firms on the right tail of the distribution dis-proportionally increase their labor demand. This force pushes up wages and reduces the labor demand of firms with medium levels of productivity. Such reallocation is stronger when there is a smaller diminishing return to skilled labor in the appeal production.

This paper is first related to the misallocation literature. Hsieh and Klenow (2009) find a sizable welfare gain if revenue productivities, i.e., log TFPR, are equalized in the firm-level datasets for China and India. Recent studies, such as Ševčík (2017) and Kehrig and Vincent (2019), investigate the within-firm across-plant misallocation of resources. Instead of focusing on the across-plant intensive margin, our paper is on the extensive margin of products.

The product margin resembles extensive margins of misallocation studied in the literature. To name a few, there are Barseghayn and DiCecio (2011), Alfaro and Chari (2014), Clementi and Palazzo (2016) on firm entry and exit, Midrigan and Xu (2014), Restuccia (2014), Boedo and Şenkal (2014) and Bento and Restuccia (2020) on transitions into the formal sector, and costly productivity improvements. Another study, Jaef (2018), also studies the distorted product margin and is closest to ours. While he models the product choice continuous, we make it binary to enable the model readily speak to moments of MP firms in firm-level datasets. We also contribute to the discussion in providing the firm-level evidence on the distorted product margin and finding a similar magnitude in the product and firm extensive margins.

Our paper is also related to the multi-product literature. Multi-product analysis was primarily in the trade literature that studies how lower trade costs are associated with more products exported by firms. See, for example, Eckel and Neary (2010), Bernard et al. (2011), Mayer et al. (2014) and Manova and Yu (2017). Outside of the trade literature, studies such as Bernard et al. (2010), Broda and Weistein (2010), Goldberg et al. (2010), Minniti and Turino (2013), Hottman, Redding, and Weistein (2016) and Argente, Lee, and Moreira (2019) investigate within- and across-firm product add-and-drop to understand firm size differences and the aggregate output fluctuation. We contribute to this literature by showing how distortions could affect firm product decisions.

The rest of the paper is structured as follows. Section 2 discusses the data and documents the stylized facts on multi-product firms in China, and the negative relation between distortions and firms' multi-product status. Section 3 presents a discrete product choice model with firm entry and exit. Section 4 calibrates the model to the data and implements counterfactual experiments. Section 5 concludes.

# 2 Data

This paper mainly uses the Annual Survey of Industrial Firms (ASIF) surveyed by the National Bureau of Statistics of China from 1998 to 2007. Although extensively studied in the literature (e.g., Hsieh and Klenow, 2009; Brandt, Van Biesebrock, and Zhang, 2014; Bai, Lu, and Tian, 2016), the product aspect of this dataset has been underexplored. Compared to the Indian Prowess data in Goldberg et al. (2010), the ASIF data covers more small- and medium-sized unlisted firms and hence suits better for the discussion on distorted firm-level product scopes in developing economies.[5]

We utilize three variables: *Main product 1*, *Main product 2* and *Main product 3* that record names of products (in Chinese characters) produced by firms each year. Here *firm* is defined as a production unit with its unique legal identity at the State Administration for Industry and Commerce, following the U.S. Bureau of Labor Statistics convention. Names of products are at the 6-digit Chinese Industrial Classification (CIC) level, closest to the 5-digit SIC product definition used in Bernard et al. (2010) and coarser than the HS8 code or the UPC code in studies such as Manova and Yu, 2017 and Broda and Weistein (2010).[6] For instance, Lenovo Beijing Limited produced personal computers (400901) as its unique product

[5]In 2004, the ASIF data covers 20% of manufacturing firms in China. This number is only about 4% in the Prowess data.

[6]Under the CIC system, industries are defined in 2-digit, 4-digit, 6-digit, and 10-digit. This paper names 6-digit as products, 4-digit as industries, and 2-digit as sectors, consistent with languages adopted in Bernard et al. (2010).

in 1999 and later added laser printers (401299) starting from 2000. Nevertheless, Lenovo's cellphone business (400401) is under a separate legal entity, Lenovo Mobile Technology Limited.

We focus on the extensive margin of firms producing single- versus multiple-product(s). We drop observations that have missing values for the three product variables (1% firm-year observations), classify firms with one product as single-product (SP) producers, and those with two and three products as multi-product (MP) producers. The choice of SP-MP margin is due to practical considerations by the maximum of three products in the data. This dichotomy, however, is sufficient to illustrate the importance of the product margin in our following empirical and quantitative analysis.

## 2.1 Fewer and Smaller Multi-Product Firms

This section presents the first stylized fact on MP firms in China: fewer and smaller than those in the U.S.

**Fewer and Smaller** Table 1 documents the fraction and output share of MP firms and their relative sizes compared to SP firms in China and the U.S. According to the ASIF data, 27% firms produce more than one product, and their sales receipts are about 44% of annual industrial output. Both numbers are smaller when we change the data into the Economic Census 2004 data, which contains ASIF firms as well as all the rest of manufacturing firms with sales less than the threshold value of ASIF. These numbers are in contrast to the more important role played by MP firms in the U.S. economy, where 39% firms produce multiple products and generate 87% of annual industrial output (Bernard et al., 2010). Our results are also in contrast to Goldberg et al. (2010) that use a similar product definition and finds 47% of firms producing multiple products in the Indian's Prowess data.

In terms of sizes, we compute the relative market share of MP firms by the

Table 1: Multi-product Firms, Number, Output Share and Size, China vs U.S.

| | China | | United States |
|---|---|---|---|
| | ASIF 98-07 | Census 2004 | Census |
| Fraction | 27.00% | 16.95% | 39.00% |
| Output share | 44.20% | 41.63% | 87.00% |
| Rel. size to single-product firms | 2.14 | 3.49 | 10.46 |

*Notes:* Relative size of multi-product firms is calculated as $\frac{\bar{Y}_{MP}}{\bar{Y}_{SP}}$, where $\bar{Y}_{MP}$ and $\bar{Y}_{SP}$ are average sales of multi- and single- product firms. Numbers for United States are from Bernard et al. (2010).

following equation

$$\frac{\bar{Y}_{MP}}{\bar{Y}_{SP}} = \frac{\text{output share of MP firms}}{\text{fraction of MP firms}} \Big/ \frac{\text{output share of SP firms}}{\text{fraction of SP firms}} \tag{1}$$

which captures the idea of how large an average MP firm is in terms of an average SP firm. In China, this number equals 2.14 in the ASIF data and 3.49 in the Census data, less than a third of the relative size of MP firms in the U.S. data.

**Measurement** We next address several measurement concerns. First, the fraction of MP firms may differ between the state-owned versus the non-state-owned sector, similarly for the exporting versus the non-exporting sector. We find that the fraction and output share of MP firms barely change when we drop exporting firms and state-owned enterprises.

Second, the fewer fact could be due to the industry composition difference between China and the U.S. For instance, manufacturing is more labor-intensive, given China's role in global value chains. We compute the fraction of MP firms within each 2-digit CIC industry and sum these fractions by the weight of U.S. industry-level receipts (Statistics of Business 2014) to compute an aggregate fraction of MP firms in the economy. This adjustment increases China's fraction of MP firms from 17% to 22% in the Economic Census 2004, still smaller than the U.S. number.

Third, products reported in the Chinese data might be coarser than the U.S. census products, which could be why we observe such a low fraction of MP firms. For instance, the manager of a firm producing "Electric Oven" (391704) and "Food Processor" (391705) may report "Kitchen Appliance" (3917) out of convenience, which masks this MP firm by our definition into a seeming SP firm in the data. If all firms did so in reporting products in the ASIF data, the comparable benchmark number in the U.S. would be 28% instead of 39%, which is the fraction of firms spanning multiple 4-digit industries in the U.S. census data (Bernard et al., 2010).[7] This is, however, still larger than the 17% in our data.

Fourth, the relative market share may not accurately describe the size of MP firms. When we use the alternative size of employment, firms with more than 500 workers in China take up 35% of total employment over 1998-2007 and 52% in the U.S. in the Statistics of Business 2014 data. Given that these firms are dominantly MP producers, we conclude that MP firms are smaller in China.[8] This observation is also consistent with studies that find richer countries have larger firms on average (e.g., Hsieh and Olken, 2014; Bento and Restuccia, 2020)

## 2.2 Distortions and the Product Margin

How do distortions affect firms' status in producing multiple products? In this section, we provide suggestive evidence that distortions lower the probability of a firm becoming an MP producer, *ceteris paribus*.

**Indirect Measures: Log TFPR** We first use the log TFPR measure for firm $i$ in industry $s$ at time $t$ in the ASIF data to proxy distortions (Hsieh and Klenow,

[7] Bernard et al. (2010) reports that 28% (10%) of U.S. firms produce in multiple 4-digit (2-digit) SIC industries, generating 81% (66%) of manufacturing receipts.

[8] In the ASIF data, 39% (33%) firms in the top 2% (5%) of the sale's distribution are MP firms.

2009)

$$
\begin{aligned}
logTFPR_{ist} &= 0.5 * log(\frac{ValueAdded}{WageBill})_{ist} + 0.5 * log(\frac{ValueAdded}{CapitalStock})_{ist} \\
&= LaborDistortion_{ist} + CapitalDistortion_{ist}
\end{aligned} \quad (2)
$$

where the capital share is 0.5. We call this measure indirect since it includes all possible distortions without specifically naming them (Restuccia and Rogerson, 2013). $LaborDistortion$ and $CapitalDistortion$ separately reflect distortions in the labor and capital markets, and each of them also indistinguishably reflects distortions in the output market.[9]

For an intuitive look at the distorted product margin, we classify firms into six types according to their MP status and whether they are subsidized, taxed with a lower or higher rate than the median tax rate. Table 2 tabulates average levels of distortions, government subsidy intensity (% of sales), borrowing rates, log gross output productivity as in Ackerberg, Caves, and Frazer (2015) and markups as in De Loecker and Warzynski (2012) for each type during 2004-2007. We choose the subsample of 2004-2007 because both materials and other intermediate inputs (mainly energy and services) are available in the ASIF data, making the productivity and markup estimation in a gross output production function viable.[10]

We first spot evidence of the distorted product margin in Table 2. Comparing the group of SP & Taxed High firms with that are MP & Subsidized, we find the former group are 20% more productive and 114% more taxed. Absent distortions, multi-product theories (e.g., Klette and Kortum, 2004; Nocke and Yeaple, 2014; Jaef, 2018) predict that firms with lower marginal costs or higher productivities equivalently are more likely to overcome fixed costs of new product lines and profit. In other words,

[9]Our model section will show that the empirical measure of distortion does not depend on the firm-level number of products when production functions are in Cobb-Douglas forms.

[10]Our markup estimates are close to an estimate of 1.41 given in De Loecker and Eeckhout (2018) for China. If we estimated a value-added production function in the 1998-2007 data, the median markup estimate would exceed 80%, which is absurdly high.

we shall expect the former group with an ACF productivity of 0.2614 to produce multiple products, not the latter group with a much lower productivity level. The contrast between theories and our results suggests the potential impact of distortions on the product numbers.

We second find the size-dependent distortion phenomenon, i.e., more productive firms face higher distortions. The correlation coefficient between the firm-level TFPR and the ACF productivity is 0.282. Across types, conditional on the SP status of firms, the average level of distortion increases from -35.20% to 83.12% from the first to the third column, and the ACF productivity monotonically increases from 0.0184 to 0.2614. The result is similar conditional on the MP status of firms. Furthermore, the size-dependent distortion is robust when we replace the log TFPR measure by other distortion measures, such as the government subsidy intensity and borrowing interest rates. More productive firms receive fewer government subsidies and are charged with higher interest rates, although they do not necessarily charge lower markups. Our results hence complement with the literature on size-dependent distortions (e.g., Guner et al., 2008; Restuccia and Rogerson, 2008; Gourio and Roys, 2014; Garcia-Santana and Pijoan-Mas, 2014).

Lastly, we confirm that conditional on the level of distortions, MP firms are indeed more productive than SP firms in our data. We mentioned that the comparison between groups of SP & Taxed High and MP & Subsidized goes against multi-product theories. However, that finding has both the productivity and the distortion measure varying. Given a similar distortion level, i.e., reading Table 2 column by column, we find MP firms are about 3.6% to 5% more productive than SP firms, which is consistent with multi-product theories.

We then proceed to a Probit regression to identify the impact of distortions on the MP probability, to quantify the distorted product margin. For firm $i$ in industry

Table 2: Product Extensive Margin by Discrete Types, 2004-2007

| | | Subsidized | Taxed | |
|---|---|---|---|---|
| | | | Low | High |
| SP | Distortion (TFPR) | -35.20% | 44.09% | 83.12% |
| | ACF log Productivity | 0.0184 | 0.1347 | 0.2614 |
| | Govern't subsidy (% of sales) | 0.16% | 0.15% | 0.11% |
| | Borrowing Rate | 1.87% | 2.12% | 2.63% |
| | DW Markup | 1.43 | 1.36 | 1.42 |
| MP | Distortion (TFPR) | -35.96% | 43.38% | 82.66% |
| | ACF log Productivity | 0.0612 | 0.1777 | 0.3109 |
| | Govern't subsidy (% of sales) | 0.21% | 0.19% | 0.15% |
| | Borrowing Rate | 1.77% | 1.94% | 2.20% |
| | DW Markup | 1.45 | 1.39 | 1.43 |

*Notes*: ASIF data 2004-2007. Numbers are average levels for each variable. The ACF log productivity is estimated using the Ackerberg et al. (2015)'s method in a gross output production function and uses the variable of other intermediate inputs (mainly energy and services) to invert the productivity. DW markup is estimated using the De Loecker and Warzynski (2012)'s method.

$s$ at time $t$, we run

$$P(Multi_{ist}) = \Phi(\beta_0 + \beta_1 Distortion_{ist} + \beta_2 Age_{ist} + \beta_3 SOE_{ist} + \\ \beta_4 Exporter_{ist} + \beta_5 LogAsset_{ist} + \delta_s + \delta_t + \epsilon_{ist}) \quad (3)$$

where $SOE_{ist}$ and $Exporter_{ist}$ are state-owned and exporter dummies, and $\delta_s$ and $\delta_t$ are industry and year fixed effects.

Table 3 presents marginal effects at means (MEM) for Probit regressions. In the baseline specification (1), a one standard deviation increase in log TFPR (0.42) decreases the MP probability by 1.18 percentage points, which is considerable given the 27% fraction of MP firms. The negative effect is also statistically significant if we replace the log TFPR by the labor and capital distortion alone (specification (2) and (3)) and by the one-year lagged log TFPR after further controlling for the lagged MP status (specification 4). In the sub-sample of 2004-2007, specification (5) further controls for markup to ensure that the negative coefficient is not caused by a lower markup charged by MP firms. We again find that the negative marginal effect similar to that of the specification (1).

Therefore, we conclude that Table 3 provides the cross-sectional evidence on the product extensive margin. Table 3 also indicates that older firms, SOE firms, exporting firms, and larger firms are more likely to be MP producers.

One may be concerned about the endogeneity of log TFPR in estimating equation (3). To alleviate the concern, we study two direct measures of distortions: corporate income tax and self-reported distortion.

**Direct Measures: Corporate Income Tax** In the first one, we extend the ASIF data to 2012 and exploit the natural experiment of 2008 corporate income tax reform. The reform aimed to stimulate market competition by eliminating the corporate tax burden difference between ordinary firms and those policy-favored ones.[11] Before

[11] The policy was announced in March 18, 2007 and implemented in January 1, 2008. The reform was one step of the agenda in gradually establishing a modern tax system in China, and shall not

Table 3: Marginal Effects of Distortions on the MP Probability, Probit

| | 1998-2007 | | | | 2004-2007 |
|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) |
| Distortion | -0.0282*** | | | | -0.0242*** |
| | (-30.92) | | | | (-14.94) |
| Labor Distortion | | -0.0543*** | | | |
| | | (-41.40) | | | |
| Capital Distortion | | | -0.0007** | | |
| | | | (-2.98) | | |
| 1 Yr Lagged Distortion | | | | -0.0111*** | |
| | | | | (-7.57) | |
| 1 Yr Lagged MP Status | | | | 0.756*** | |
| | | | | (447.85) | |
| DW Markup | | | | | -0.0161*** |
| | | | | | (-8.17) |
| Age | 0.0040*** | 0.0042*** | 0.0042*** | 0.00100*** | 0.0026*** |
| | (111.00) | (111.75) | (115.40) | (17.93) | (40.68) |
| SOE | 0.0159*** | 0.0159*** | 0.0163*** | -0.0029 | 0.0256*** |
| | (14.72) | (14.77) | (15.13) | (-1.72) | (11.63) |
| Exporter | 0.0271*** | 0.0241*** | 0.0281*** | 0.0090*** | 0.0248*** |
| | (31.38) | (27.70) | (32.48) | (6.84) | (19.69) |
| Log Asset | 0.0392*** | 0.0419*** | 0.0401*** | 0.0157*** | 0.0343*** |
| | (142.94) | (152.97) | (142.05) | (36.97) | (89.76) |
| Industry FE | YES | YES | YES | YES | YES |
| Year FE | YES | YES | YES | YES | YES |
| N | 1521750 | 1521750 | 1521750 | 908184 | 640420 |
| Prob(Correctly Classified) | 74.13% | 74.12% | 74.09% | 93.83% | 71.87% |

t statistics in parentheses

* $p<0.05$ ** $p<0.01$ *** $p<0.001$

*Notes*: ASIF data 1998-2007. Observations with the distortion (TFPR) measure greater than 1 or smaller than -1 are dropped. The ACF log productivity is estimated using the Ackerberg et al. (2015)'s method in a gross output production function and uses the variable of other intermediate inputs (mainly energy and services) to invert the productivity. DW markup is estimated using the De Loecker and Warzynski (2012)'s method. Results are marginal effects at the means.

2008, Small Enterprises, High Tech Enterprises, firms in West Development Program, and Economic Zones have corporate income tax rates at a maximum of 25%, lower than the baseline rate of 33%. Starting from 2008, the baseline rate decreased to 25%, while rates in these programs remained largely unchanged (see details in Appendix). Presumably, firms outside of these programs were less distorted after 2008 than before, and more likely to produce multiple products, *ceteris paribus*.

To verify this idea, we create a *balanced* panel of firms that spans from 2004 to 2012 (excluding 2010).[12] The balanced structure fixes the same set of firms over time to avoid policy arbitrages of new firms.[13] We run the following regression in spirit of equation (3)

$$P(Multi_{ist}) = \Phi(\beta_0 + \beta_1 Treat_{is} + \beta_2 Post08_t + \beta_3 Treat_{is} * Post08_t + \beta_4 Age_{ist} + \beta_5 SOE_{ist} + \beta_6 logAsset_{ist} + \delta_s + \delta_t + \epsilon_{ist}) \quad (4)$$

where the treatment group is defined as firms outside of pre-08 favored programs before 2008, vice versa for the control group. Our difference-in-difference estimator $\hat{\beta}_3$ thus illustrates the additional post-08 change in the fraction of MP firms in the treatment group compared to the control group.

Table 4 presents estimates of marginal effects at the means for the Probit regression. In column (1), the gap of MP probabilities between the treatment group and the control group is -0.0379 before 2008. This is consistent with the obesrvation that treated firms were charged with higher tax rates. Our difference-in-difference estimate, $\hat{\beta}_3$, is 0.0120 and statistically significant. Thus, the gap of MP probabilities between the two groups narrow by about 32% (0.0120/0.0379) after 2008, consistent with the idea that a lower corporate income tax rate is associated with a higher MP

be viewed as a preemptive policy for the 2008 financial crisis.

[12]We drop the year 2010 because of its data irregularity, following the literature (e.g., Chen, Chen, Liu, Suárez Serrato, and Xu, 2021)

[13]We start our panel early enough to avoid strategic behaviors of firms switching groups, although the incentive is unlikely given the similar corporate tax rate between the two groups after 2008.

probability.

One confounding factors in estimating equation (4) is the 2008 crisis. It could be the case that firms in the control group exported more and reduced the number of products due to the negative global demand shock. This mechanism would bias the coefficient of $\hat{\beta}_3$ upwards. To overcome this problem, we split the data into the non-exporter and exporter sub-samples and separately estimate equation (4). Firm in the non-exporter sub-sample is relatively insulated from the negative global shock and thus the estimate $\hat{\beta}_3$ shows a clean difference-in-difference effect of tax reforms.

Column (2) shows similar results for the non-exporter sub-sample, compared to those in column (1). Within this sub-sample, treated firms are 4.58% less likely to be MP producers before 2008. This gap narrows by a robust 32% (0.0147/0.0458) significantly after 2008. Meanwhile, the probability of correct classification increases from 70.87% to 72.02%, suggesting a better explanatory power of the model in predicting the MP probability for non-exporters.

For the exporter sub-sample, column (3) suggests a mixed effect of tax reforms and the 2008 crisis. Treated firms are still less likely to be MP producers before 2008, but the magnitude and the t-statistic suggest a negligible difference, compared to the controlled ones. The difference-in-difference estimate is negative but insignificant. Comparison between column (2) and (3) indicates the importance of isolating the crisis shock from the tax reform shock.

**Direct Measures: Self-Reported Distortions** We further bolster our argument of the distorted product margin using the self-reported distortion measures using the World Bank Enterprise Survey (WBES) 2012 data. In the survey, firms are asked if they face obstacles in access to finances, transport, custom and trade regulations, electricity and telecommunications, access to land, labor regulations, and others (see Figure A2 in Appendix for the detailed questions from the WBES questionnaire). We set distortion dummies 1 if firms report moderate or more severe obstacles for each aspect as listed in Table 5, and 0 otherwise.

On the product side, unlike the ASIF data, the WBES data does not provide the product information of firms. We derive the MP dummy from the following question: What percentage of total sales does the main product represent? We define a firm to be MP if this percentage exceeds 90%.[14] This criterion gives the fraction of MP firms 30%, closest to that in the ASIF data.

We then run a similar Probit regression as in equation (3). The distortion variable is included one at a time, again controlling for whether the firm is an exporter, state-owned enterprise, age and size. We define a state-owned firm when it has a positive state equity share. By the lack of total asset, the size of a firm is defined by which bin of employment, i.e., 5 to 19, 20 to 99, or 100 and above, the firm is in.

Table 6 shows that all distortions except for that of labor have a negative impact on the probability of being a MP firm. Specifically, distortions on finance and on transport, custom and trade regulations, i.e., regulations that prohibit firms geographically expanding their product market, are significantly negatively associated with firms' product scopes. Quantitatively, if a firm's financial obstacle switches from 0 to 1, its MP probability decreases by 5.65 percentage points. Similarly, if its obstacle in transport, custom and trade regulations switches from 0 to 1, its MP probability decreases by 9.94 percentage points. These numbers are sizable compared to the fact that only 30% firms are MP producers in the WBES data.

For other distortion measures, most of them negatively affect a firm's MP probability, although statistically insignificant. The labor distortion is an exception and has a positive but insignificant impact on the MP probability.[15] We thus view results

[14] We tried a different level of 95%. Results are similar to what follows.

[15] This result may look contradictory to the negative and statistically significant coefficient in column (2) of Table 3. This difference may arise from (i) there are labor distortions other than those mentioned in the WBES; (ii) labor distortions defined in equation (2) may reflect output market distortions, rather than that in the labor market. In fact, if we regress the MP dummy on the labor distortion calculated in the WBES data using equation (2), we still find a negative coefficient.

Table 4: DID Marginal Effects of Tax Reduction on the MP Probability

| | (1) | (2) | (3) |
|---|---|---|---|
| | All | Non-Exporter | Exporter |
| Post08 | -0.0488*** | -0.0598*** | -0.0132 |
| | (-13.30) | (-14.54) | (-0.0132) |
| Treat | -0.0379*** | -0.0458*** | -0.0068 |
| | (-16.50) | (-18.04) | (-1.29) |
| Treat*Post08 | 0.0120*** | 0.0147*** | -0.0025 |
| | (3.85) | (4.23) | (-0.35) |
| Exporter | 0.0346*** | | |
| | (22.83) | | |
| SOE, Age, Size Controls | YES | YES | YES |
| Industry FE | YES | YES | YES |
| Year FE | YES | YES | YES |
| N | 511,584 | 382,353 | 129,231 |
| Prob(Correctly Classified) | 70.87% | 72.02% | 67.54% |

t statistics in parentheses

* p<0.05, ** p<0.01, *p<0.001

*Notes:* ASIF data 2004-2012 (excluding 2010). *Treat* dummies are 1 if firms were outside of policy favored programs before 2008. *Exporter* dummies are 1 if firms exported before 2008. Results are marginal effects at the means.

Table 5: Distortion Measures in the WBES 2012 Data

| Distortion | Equals to 1 with a moderate, major, or very severe obstacle in |
|---|---|
| FIN | Access to finance |
| TCT | Transport, custom and trade regulations |
| ET | Electricity, telecommunications |
| LAND | Access to land |
| LABOR | Labor regulations, inadequately educated workforce |
| OTH | Practices of competitors in the informal sector; Crime, theft and disorder; Tax rates, tax administration, business licensing and permits, political instability, corruption, courts |

*Notes*: WBES data 2012. Distortion dummies are defined by authors.

Table 6: Marginal Effects of Distortions on the MP Probability, Probit, WBES Data

| | FIN | TCT | ET | LAND | LABOR | OTH |
|---|---|---|---|---|---|---|
| Distortion | -0.0565* | -0.0994** | -0.0640 | -0.0340 | 0.0417 | -0.0064 |
| | (-2.01) | (-2.71) | (-1.84) | (-1.03) | (1.56) | (-0.29) |
| Exporter, SOE, Age, Size Controls | YES | YES | YES | YES | YES | YES |
| Industry FE | YES | YES | YES | YES | YES | YES |
| N | 1645 | 1598 | 1656 | 1653 | 1656 | 1659 |

t statistics in parentheses

* p<0.05 ** p<0.01 *** p<0.001

*Notes*: WBES data 2012. SOE is defined as 1 if the firm has a positive state equity share. Without the total asset variable in the WBES data, the *Size* variable controls for the size effect, and equals to 1 if the employment is from 5 to 19, 2 if it is from 20 to 99, and 3 if it is larger than 100. Results at marginal effects at the means.

in Table 6 further bolster our conclusion about the negative effect of distortions on the product margin in the ASIF data.

**Summary** Section 2 documents several regularities on the MP firms in a developing economy like China. First, MP firms are fewer and smaller compared to the U.S. Second, using indirect and direct measures of distortions, we robustly find that the MP probability of firms decreases when the level of distortions increases.

# 3 Model

This section builds discrete product decisions of firms into a general equilibrium model with entry and exit à la Melitz (2003). In order to capture the distorted product margin in the cross-sectional data in Section 2, we set firm-level productivities and product-level appeal shocks permanent, different from Bernard et al. (2010) and Jaef (2018). We set distortions as a revenue output tax that is increasing in firm-level productivities (i.e., size-dependent). We will show how distortions alter the productivity cutoffs of firms above which they produce multiple products.

## 3.1 Demand

We assume that the product demand arises from a representative household who supplies unskilled labor $L$ and skilled labor $H$ inelastically. The household maximizes utility from a continuum of goods:

$$U = \left[ \int_{\omega \in \Omega} (\lambda(\omega) q(\omega))^{\rho} \, d\omega \right]^{\frac{1}{\rho}}, \; 0 < \rho < 1 \tag{5}$$

where $\omega$ represents a variety, $\Omega$ is the endogenous set of firm-product varieties in the economy, and $\rho$ determines the elasticity of substitution across varieties $\sigma =$

This continuous labor distortion variable is about 2% higher not only if the labor distortion dummy $LABOR = 1$ but also if the transportation dummy $TCT = 1$, confirming our conjecture (ii).

$1/(1-\rho)$. We assume that firms and products are measure zero. The product appeal $\lambda(\omega)$ (e.g., quality or taste) captures how the representative household values the quantity consumed $q(\omega)$. According to Hottman et al. (2016), unobserved product appeals beyond price factors explain more than 50% of the cross-sectional variation of U.S. firm sizes. Therefore, instead of a Bertrand competition model of product innovations (Klette and Kortum, 2004), we model similarly to Bernard et al. (2010) that embed the appeal into the household's utility function.

The expenditure minimization problem gives the demand and revenue for each product:

$$q(\omega) = \frac{Q}{\lambda(\omega)}\left[\frac{p(\omega)}{\lambda(\omega)P}\right]^{-\sigma}, \; r(\omega) = R\left[\frac{p(\omega)}{\lambda(\omega)P}\right]^{1-\sigma} \tag{6}$$

where $Q \equiv U$ is the appeal-adjusted aggregate consumption index, and $P$ denotes the associated aggregate price index:

$$P = \left[\int_{\omega\in\Omega}\left(\frac{p(\omega)}{\lambda(\omega)}\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} \tag{7}$$

The aggregate revenue $R$ is defined as $R = PQ$.

## 3.2 Production Settings

Firms are identical prior to entry. Upon entry, they pay a fixed cost, $f_e > 0$, to draw a productivity $\varphi$ from a common cumulative distribution function $G(\varphi)$. Additionally, they draw a pair of appeal shocks $a_1$ and $a_2$ for two product lines. Each shock is independently drawn from a Bernoulli distribution with a probability of a high realization $a_h$ being $p$, and that of a low realization $a_l$ being $1-p$. Each firm's productivity and appeal shocks are assumed to be permanent. At the end of each period, incumbents exogeneously exit with the rate of $\delta$.

Firms hire skilled and unskilled labor. The quantity of each product $i$ is produced by the unskilled labor $l$ with a linear production function: $q_i = \varphi l_i$ , for $i = 1, 2$.

Across products, the firm-level productivity $\varphi$ is common and labor is divisible. Meanwhile, the product appeal $\lambda_i$ is produced by combining the skilled labor input $h$ and the exogenous shock $a_i$ with a decreasing return to scale parameter $\alpha$, i.e., $\lambda_i = a_i h_i^{\alpha}$, $i = 1, 2$. This simple functional form captures the idea that with more human capital, firms produce better quality products (Stokey, 1991) and better-seller products with a better market understanding (Gourio and Rudanko, 2014). The size of $\alpha$ hence affects the relative importance of skilled labor in productions.

We consider a stationary environment, in which the aggregate quantity $Q$, price index $P$, measure of firm $M$ and wages $\{w_h, w_l\}$ for skilled and unskilled labor are constant over time. We introduce distortions $\tau$ in the form of a proportional revenue tax if $\tau > 0$, or a subsidy in the case if $\tau < 0$. Like the productivity, the distortion is imposed at the firm-level and common for both products. Given the aggregate quantity, price, wages, productivity, distortions, and product-level appeal shocks, firms choose labor demands, the number of products to produce, their quantities, qualities and prices to maximize the profit:

$$\max_{\{l_i\}_{i=1}^n, \{h_i\}_{i=1}^n, \{q_i\}_{i=1}^n, \{p_i\}_{i=1}^n, n \in \{1,2\}} [(1-\tau) p_1 q_1 - w_l l_1 - w_h h_1 - w_l f_0 - w_l f_1 + \mathbb{1}((1-\tau) p_2 q_2 - w_l l_2 - w_h h_2 - w_l f_2)] \quad (8)$$

subject to the inverse demand curve $q_i = \frac{Q}{\lambda_i} \left[\frac{p_i}{\lambda_i P}\right]^{-\sigma}$, the appeal production technology $q_i = \varphi l_i$ and the quality production technology $\lambda_i = a_i h_i^{\alpha}$, $i = 1, 2$. Note that in addition to the variable labor cost, firms need to pay a headquarter cost $f_0$, and a fixed cost for each product $f_i$, $i = 1, 2$, in units of unskilled labor.

By the divisible labor and output across products, the optimal pricing rule is $p_i = \frac{w_l}{(1-\tau)\rho\varphi}$. The quantity for product $i$ is

$$q_i = (1-\tau)^{\frac{\sigma}{1-\alpha(\sigma-1)}} a_i^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} Q^{\frac{1}{1-\alpha(\sigma-1)}} P^{\frac{\sigma}{1-\alpha(\sigma-1)}} \left(\frac{\alpha(1-\frac{1}{\sigma})}{w_h}\right)^{\frac{\alpha(\sigma-1)}{1-\alpha(\sigma-1)}} \left(\frac{w_l}{\rho\varphi_i}\right)^{\frac{\alpha(\sigma-1)-\sigma}{1-\alpha(\sigma-1)}}. \quad (9)$$

and the appeal is

$$\lambda_i = (1-\tau)^{\frac{\alpha\sigma}{1-\alpha(\sigma-1)}} a_i^{\frac{1}{1-\alpha(\sigma-1)}} Q^{\frac{\alpha}{1-\alpha(\sigma-1)}} P^{\frac{\alpha\sigma}{1-\alpha(\sigma-1)}} \left(\frac{\alpha(1-\frac{1}{\sigma})}{w_h}\right)^{\frac{\alpha}{1-\alpha(\sigma-1)}} \left(\frac{w_l}{\rho\varphi_i}\right)^{\frac{-\alpha(\sigma-1)}{1-\alpha(\sigma-1)}}. \tag{10}$$

Hence, the firm-level profit is

$$\begin{aligned}\pi(\varphi, a_1, a_2, \tau) &= \kappa(1-\tau)\left(p_1 q_1 + \mathbb{I} p_2 q_2\right) - w_l(f_0 + f_1 + \mathbb{I} f_2) \\ &= \kappa(1-\tau) r(\varphi, a_1, a_2, \tau) - w_l(f_0 + f_1 + \mathbb{I} f_2) \end{aligned} \tag{11}$$

where $\kappa = 1 - (1-\frac{1}{\sigma})(1+\alpha)$ and $r(\varphi, a_1, a_2, \tau)$ is the firm-level revenue

$$r(\varphi, a_1, a_2, \tau) = (1-\tau)^{\frac{(1+\alpha)(\sigma-1)}{1-\alpha(\sigma-1)}} \left(a_1^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} + \mathbb{I} a_2^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}\right) B\varphi^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} \tag{12}$$

with $B = \left(RP^{\sigma-1}\right)^{\frac{1}{1-\alpha(\sigma-1)}} \left[\frac{\alpha(1-\frac{1}{\sigma})}{w_h}\right]^{\frac{\alpha(\sigma-1)}{1-\alpha(\sigma-1)}} \left(\frac{\rho}{w_l}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}$.

To insure a positive revenue, we assume $\kappa > 0$, i.e.,

**Assumption 3.1.** $(1-\frac{1}{\sigma})(1+\alpha) < 1$.

Equations (9)-(12) suggest that the product quantity $q_i$, appeal $\lambda_i$, profit $\pi$ and revenue $r$ are decreasing in the revenue tax distortion $\tau$, increasing in the appeal shock $a_i$ and productivity $\varphi_i$. When the economy has a higher human capital endowment $H$, the total welfare increases as the wage of skilled labor $w_h$ decreases and the quality $\lambda$ for each variety $\omega$ increases.

We illustrate the product extensive margin in an example with two firms, firm 1 and 2, which are characterized by $(\varphi, a_1, a_2, \tau)$and $(\varphi', a_1', a_2', \tau')$, where $\varphi > \varphi', a_1 > a_1', a_2 > a_2', \tau > 0, \tau' < 0$. The relative quantity between the two firms is

$$\frac{q(\varphi,\tau)}{q(\varphi',\tau')} = \frac{a_1^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} + \mathbb{I} a_2^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}}{a_1'^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} + \mathbb{I} a_2'^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}} \left(\frac{1-\tau}{1-\tau'}\right)^{\frac{\sigma}{1-\alpha(\sigma-1)}} \left(\frac{\varphi}{\varphi'}\right)^{\frac{\sigma-\alpha(\sigma-1)}{1-\alpha(\sigma-1)}} \tag{13}$$

In a multi-product framework, the level of misallocation could be enlarged if the numerator firms is taxed into a SP firm, and the denominator firm is subsidized into a MP firm. This effect is larger when the role of human capital $\alpha$ and the elasticity of substitution $\sigma$ increases.

We next introduce a span of control parameter $\xi \in [0, 1]$ into the model through the appeal production for the second product, i.e., $\lambda_2 = \xi a_2 h_2^{\alpha}$. A value of $\xi < 1$ indicates that the revenue of the second product decreases relative to the first, capturing the decreasing return on the dimension of products with a limited management capacity (Caliendo and Rossi-Hansberg, 2012; Gibbons and Roberts, 2013). Such a declining productivity ladder along products is also a key feature of models in Eckel and Neary (2010) and Mayer et al. (2014). We will show that the difference in $\xi$ between China and U.S. helps pin down relative sizes of multi-product firms, and intuitively reflects the cross-country difference of management practices (Bloom and Van Reenen, 2007; Bloom et al., 2020).[16] With this $\xi$, all previous equations still hold with a modified $a_2$ to be now $\xi a_2$.

A feature of our model is the capability of backing out distortion from firms' balance sheet data, irrespective of the number of products. Specifically, the total revenue over the wage bill is

$$\frac{p_1 q_1 + \mathbb{I} p_2 q_2}{w_l l + w_h h} = \frac{1}{(1-\tau)(1-\frac{1}{\sigma})(1+\alpha)} \tag{14}$$

## 3.3 Decision on the Number of Products

Different from the single-product setting, firms here decide the number of products to produce. We will show that the optimal product-line decision follows the form of cutoff strategies, similar to Melitz (2003).

Motivated by the size-dependent distortion in Restuccia and Rogerson (2008) and

[16] Another interpretation of $\xi$ is the product-specific distortion, which unfortunately cannot be uncovered absent product-level output and input information in the data. We prefer the span of control narrative since we are not sure why possibly Chinese firms could be subject to an increasing product-specific distortion when the number of products increases while their U.S. counterparts are not.

Guner et al. (2008), we introduce a simple and stylized specification of $\tau$ for each entering firm[17]

$$\tau = \begin{cases} \tau_t > 0 & \text{if } \varphi > \varphi_{tax} \\ 0 & \text{if } \varphi \in [\varphi_{sub}, \varphi_{tax}] \\ \tau_s < 0 & \text{if } \varphi < \varphi_{sub} \end{cases} \tag{15}$$

We also impose the following two assumptions on the per-product fixed cost.

**Assumption 3.2.** *The fixed cost of producing the second product $f_2$ is sufficiently large compared to the fixed cost of producing only one product $f_0 + f_1$:*

$$min\{(\frac{a_h}{a_l})^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}, \xi^{-\frac{\sigma-1}{1-\alpha(\sigma-1)}}\} > \frac{f_0 + f_1}{f_2}$$

**Assumption 3.3.** *The per-product fixed costs are equal, i.e., $f_1 = f_2$*

Under Assumption 3.2, the cutoff for a firm above which it produces two products is greater than the cutoff for producing one product, regardless of the level of appeal shocks and distortions (see the proof in Appendix). In other words, if the fixed cost of a second product is $\epsilon$-small, any firm produces two products, which contradicts the empirical observation that multi-product firms are on average more productive. Without loss of generality, Assumption 3.3 further simplifies our analysis.

A subsidy-like distortion lowers the cutoffs for being profitable while a tax-like distortion increases them. We denote the cutoffs with a superscript star in the economy with distortions and add superscript $\{t, s, 0\}$ to denote the cutoffs for firms with tax-like, subsidy-like, and zero distortions. When the economy is free of distortions, the cutoffs are denoted with a double-star superscript.

We illustrate how to derive the cutoffs using the example of firms with appeal shocks $(a_h, a_l)$. Their cutoff is $\varphi_{hl,2}^{*j}$ for producing the second product, and $\varphi_{hl,1}^{*j}$ for producing the first product, for $j \in \{t, s, 0\}$. With zero-profit conditions, the

[17] In our quantitative analysis, we also consider the specification where $1 - \tau(\varphi) = \varphi^{-\gamma}, \gamma > 0$ as in Jaef (2018).

first-product cutoff $\varphi^{*j}_{hl,1}$ is determined by

$$\kappa(1-\tau_j)^{\frac{\sigma}{1-\alpha(\sigma-1)}} B \left(a_h \varphi^{*j}_{hl,1}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} = w_l(f_0+f_1) \tag{16}$$

and the second-product cutoff $\varphi^{*j}_{hl,2}$ is determined by

$$\kappa(1-\tau_j)^{\frac{\sigma}{1-\alpha(\sigma-1)}} B \left(\xi a_l \varphi^{*j}_{hl,2}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} = w_l f_2 \tag{17}$$

For firms with different distortion draws, we show the following proposition

**Proposition 3.4.** *The set of products firms produce $\Omega^{hl}_{\varphi\tau}$ with productivity $\varphi$, appeal shocks $(a_h, a_l)$ and*

*(1) a tax-like distortion, $\tau > 0$ (i.e., $\varphi > \varphi_{tax}$), is*

$$\Omega^{hl}_{\varphi\tau} = \begin{cases} \{a_h, a_l\} & \text{if } \varphi > max\{\varphi_{tax}, \varphi^{*t}_{hl,2}\} \\ \{a_h\} & \text{if } max\{\varphi_{tax}, \varphi^{*t}_{hl,1}\} < \varphi \leq max\{\varphi_{tax}, \varphi^{*t}_{hl,2}\} \\ \emptyset & \text{otherwise} \end{cases} \tag{18}$$

*(2) a zero distortion, $\tau = 0$ (i.e., $\varphi \in [\varphi_{sub}, \varphi_{tax}]$), is*

$$\Omega^{hl}_{\varphi\tau} = \begin{cases} \{a_h, a_l\} & \text{if } \varphi \in [min\{\varphi_{tax}, \varphi^{*0}_{hl,2}\}, \varphi_{tax}] \\ \{a_h\} & \text{if } \varphi \in [max\{\varphi_{sub}, \varphi^{*0}_{hl,1}\}, min\{\varphi_{tax}, \varphi^{*0}_{hl,2}\}] \\ \emptyset & \text{otherwise} \end{cases} \tag{19}$$

*(3) a subsidy-like distortion, $\tau < 0$ (i.e., $\varphi < \varphi_{sub}$), is*

$$\Omega^{hl}_{\varphi\tau} = \begin{cases} \{a_h, a_l\} & \text{if } \varphi \in [min\{\varphi_{sub}, \varphi^{*s}_{hl,2}\}, \varphi_{sub}] \\ \{a_h\} & \text{if } \varphi \in [\varphi^{*s}_{hl,1}, min\{\varphi_{sub}, \varphi^{*s}_{hl,2}\}] \\ \emptyset & \text{otherwise} \end{cases} \tag{20}$$

*Similar results hold for firms with appeal shocks $(a_h, a_h)$ and $(a_l, a_l)$ with associated cutoffs. See proofs in Appendix.*

Note that by equation (16), firms producing one product with appeal shocks $(a_h, a_h)$ and $(a_h, a_l)$ have the same cutoff, i.e., $\varphi^{*j}_{hh,1} = \varphi^{*j}_{hl,1}$, which we simplify to

$\varphi^{*j}_{h,1}$, $j \in \{t, s, 0\}$. We then show that other cutoffs can be conveniently expressed as linear functions of this benchmark cutoff (see proofs in Appendix.)

$$\left(\frac{\xi \varphi^{*j}_{hh,2}}{\varphi^{*j}_{h,1}}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} = \frac{f_2}{f_0+f_1} \tag{21}$$

$$\left(\frac{\xi a_l \varphi^{*j}_{hl,2}}{a_h \varphi^{*j}_{h,1}}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} = \frac{f_2}{f_0+f_1} \tag{22}$$

$$\left(\frac{a_l \varphi^{*}_{ll,1}}{a_h \varphi^{*}_{h,1}}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} = 1 \tag{23}$$

and

$$\left(\frac{\xi a_l \varphi^{*j}_{ll,2}}{a_h \varphi^{*j}_{h,1}}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} = \frac{f_2}{f_0+f_1} \tag{24}$$

### 3.4 Aggregation in the Stationary Equilibrium

Given the cutoff strategies, we next solve the free-entry condition, the equilibrium cutoffs, mass of firms, the aggregate price index and welfare.

For firms with appeal shocks $(a_h, a_l)$ and distortion type $j \in \{t, s, 0\}$, the average productivity for firms producing two products is

$$\tilde{\varphi}^{j}_{hl,2} = \left(\int_{\varphi^{*j}_{hl,2}}^{\infty} \left(a_h^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} + (\xi a_l)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}\right) \varphi^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} \frac{g(\varphi)}{1-G(\varphi^{*j}_{hl,2})} d\varphi\right)^{\frac{1-\alpha(\sigma-1)}{\sigma-1}} \tag{25}$$

and for those producing one product is

$$\tilde{\varphi}^{j}_{hl,1} = \left(\int_{\varphi^{*j}_{hl,1}}^{\varphi^{*j}_{hl,2}} (a_h \varphi)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} \frac{g(\varphi)}{G(\varphi^{*j}_{hl,2}) - G(\varphi^{*j}_{hl,1})} d\varphi\right)^{\frac{1-\alpha(\sigma-1)}{\sigma-1}} \tag{26}$$

We next link average profits with the average productivites as follows

$$\bar{\pi}^j_{hl,2} = \left[ \frac{f_0 + f_1}{f_0 + f_1 + f_2} \left( \frac{\tilde{\varphi}^j_{hl,2}}{a_h \varphi^{*j}_{hl,1}} \right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} - 1 \right] (f_0 + f_1 + f_2) \tag{27}$$

$$\bar{\pi}^j_{hl,1} = \left[ \left( \frac{\tilde{\varphi}^j_{hl,1}}{a_h \varphi^{*j}_{hl,1}} \right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} - 1 \right] (f_0 + f_1) \tag{28}$$

Therefore, the average profit for firms with appeal shocks $(a_h, a_l)$ is a weighted-average of $\bar{\pi}^j_{hl,1}$ and $\bar{\pi}^j_{hl,2}$

$$\bar{\pi}^j_{hl} = \frac{G(\varphi^{*j}_{hl,2}) - G(\varphi^{*j}_{hl,1})}{1 - G(\varphi^{*j}_{hl,1})} \bar{\pi}^j_{hl,1} + \frac{1 - G(\varphi^{*j}_{hl,2})}{1 - G(\varphi^{*j}_{hl,1})} \bar{\pi}^j_{hl,2}. \tag{29}$$

The same algebra holds for firms with $(a_h, a_h)$ and $(a_l, a_l)$ shocks. After taking all groups together, the average profit for all active firms can be expressed as

$$\bar{\pi} = \sum_{j=t,s,0} \left[ \frac{p^2 \left(1 - G(\varphi^{*j}_{hh,1})\right)}{p_{in}} \bar{\pi}^j_{hh} + \frac{(1-p)^2 \left(1 - G(\varphi^{*j}_{ll,1})\right)}{p_{in}} \bar{\pi}^j_{ll} + \frac{2p(1-p) \left(1 - G(\varphi^{*j}_{h,1})\right)}{p_{in}} \bar{\pi}^j_{hl} \right] \tag{30}$$

The free-entry condition for potential entrants is thus equating this average profit to the entry cost, i.e.,

$$p_{in} \frac{\bar{\pi}}{\delta} = w_l f_e \tag{31}$$

where $p_{in}$ is the survival probability for an entry firm:

$$p_{in} = \sum_{j=t,s,0} \left[ p^2 \left(1 - G(\varphi^{*j}_{hh,1})\right) + 2p(1-p) \left(1 - G(\varphi^{*j}_{h,1})\right) + (1-p)^2 \left(1 - G(\varphi^{*j}_{ll,1})\right) \right]$$

Therefore, equation (30) and (31) solve $\{\bar{\pi}, \varphi^*_{h,1}\}$. After we solve $\varphi^*_{h,1}$, we can solve other cutoffs. When the economy is free of distortions, we have $\varphi^{*s}_{a_1,a_2} = \varphi^{*t}_{a_1,a_2} = \varphi^{**}_{a_1,a_2}$ for any appeal shocks $(a_1, a_2)$.

The stationary equilibrium requires that the measure of firms enter and exit equals $\delta M = p_{in} M_e$ and the entry cost is paid by $w_l L_e = w_l M_e f_e$. Together, we have an expression for the total entry cost $w_l L_e = \frac{\delta M}{p_{in}} w_l f_e$. Using the free entry condition $p_{in}\frac{\bar{\pi}}{\delta} = w_l f_e$, we know $w_l L_e = M\bar{\pi} = \Pi$.

When there are distortions, the total distortion is $T = \int \tau pq d\omega$. We assume that $T$ is not rebated to the household, because tax- and subsidy-like distortions go beyond the real world corporate tax and subsidy. Examples include shadow values of financing, of expanding markets geographically by the lack of infrastructure, and of entry barriers that we discussed in Section 2. Under this assumption, the household budget constraint is $PQ = w_l L_p + w_h H + \Pi$. Combining this with $\Pi = w_l L_e$ and the unskilled labor market clearing condition $L = L_p + L_e$, we have $R = M\bar{r} = w_l L + w_h H$.

We assume that skilled labor supply $H$ is exogenously given, and its market-clearing condition is

$$\begin{aligned} w_h H &= \alpha(1 - \frac{1}{\sigma}) \int (1-\tau) p_i q_i di \\ &= \alpha(1 - \frac{1}{\sigma}) M\tilde{r} \end{aligned}$$

where $\tilde{r} = \sum_{j=s,t,0} \sum_{k=hh,hl,ll} (1-\tau_j) \left[ \frac{M^j_{k,1}}{M} \bar{r}^j_{k,1} + \frac{M^j_{k,2}}{M} \bar{r}^j_{k,2} \right]$ is the distortion augmented average revenue and $M^j_{k,n}$ are the measures of firms for each group, $n \in \{1, 2\}$. From the expression for the aggregate revenue $R = M\bar{r} = w_l L + \alpha(1 - \frac{1}{\sigma}) M\tilde{r}$, the measure of firms in equilibrium is $M = \frac{w_1 L}{\bar{r} - \alpha(1-\frac{1}{\sigma})\tilde{r}}$ and the wage rate for the skilled labor is $w_h = \frac{1}{H}\alpha(1 - \frac{1}{\sigma}) M\tilde{r}$.

The aggregate price index is a function the distortion adjusted average $\tilde{\varphi}$ for each group,

$$\begin{aligned} P^{\frac{1-\sigma}{1-\alpha(\sigma-1)}} &= D \int_{\omega \in \Omega} (1-\tau)^{\frac{(1+\alpha)(\sigma-1)}{1-\alpha(\sigma-1)}} \left( \xi(\omega) a(\omega) \varphi(\omega) \right)^{\frac{(\sigma-1)}{1-\alpha(\sigma-1)}} d\omega \\ &= DM \sum_{j=s,t,0} \sum_{k=hh,hl,ll} (1-\tau_n) \left[ \frac{M^j_{k,1}}{M} \tilde{\varphi}_{k,1}^{\frac{(\sigma-1)}{1-\alpha(\sigma-1)}} + \frac{M^j_{k,2}}{M} \tilde{\varphi}_{k,2}^{\frac{(\sigma-1)}{1-\alpha(\sigma-1)}} \right] \end{aligned}$$

where $D = \rho^{\frac{(\sigma-1)}{1-\alpha(\sigma-1)}} \left[ \frac{\alpha(1-\frac{1}{\sigma})}{w_h} R \right]^{\frac{\alpha(\sigma-1)}{1-\alpha(\sigma-1)}}$. The welfare per capita is $Welfare = \frac{Q}{L+H} =$

$\frac{R}{P(L+H)}$.

# 4 Quantitative Analysis

This section aims to apply our model and quantify the distorted product margin in generating welfare loss disciplined by our firm-level data. We first calibrate the model to match moments in both economies of China and the U.S. We then compute the welfare loss in our benchmark model induced by distortions. In the end, we quantify the product margin of misallocation by decomposing the welfare loss in the benchmark model via counterfactual experiments.

## 4.1 Calibration

There are 18 parameters in our model. Our calibration strategy is to first set 10 parameters either by normalization or from exogenous data sources. We then calibrate 4 parameters by matching our distortion-free model to the U.S. economy. We lastly calibrate the span of control parameter and 3 distortion-related parameters to match the Chinese economy.

**Exogenous Parameters** Table 7 summarizes exogenously chosen parameters. We set $\sigma = 4.5$, which is within the range of 4 to 10 in Broda and Weistein (2010). The distribution of productivity $\varphi$ is assumed to be normally distributed $\varphi \sim N(\mu_\varphi, \sigma_\varphi^2)$. We set the mean $\mu_\varphi = 100$ as normalization and calibrate the variance $\sigma_\varphi^2$ for the distortion-free economy. $\mu_\varphi$ can be normalized here because it is a scaling parameter and our targeted moments are all in firm shares. Even though we choose a "thin" tail distribution for productivity $\varphi$, the multi-product setting with appeal productions in our model will generate a fat tail distribution of revenue.

We set the high appeal shock $a_h = 1$. First, a different $a_h$ is isomorphic to a re-scaling of cutoff productivity $\varphi^*$ (see equation (9) and (10)). Second, other cutoff productivities are determined only by the ratio of appeal shocks $a_l/a_h$ (see equations (21) to (24)). Hence, for simplicity, we set $a_h = 1$.

For fixed costs of production, we again set $f_1 = f_2 = 100$ for normalization. Note that the profit function for each type of firm is linear in fixed costs. Combined with the free-entry condition, the model shows that it is the relative entry cost, i.e., $f_e/f_1$, that matters for the entry and production of firms. For the headquarter cost $f_0$, we are not able to separately calibrate it by the lack of product-level profit information in the data. Hence we set it to be 0.1, which is sufficiently small to ensure Assumption 3.2 to hold.

For the exogenous death rate, we set $\delta = 8\%$ to match the firm exit rate in the U.S. (Decker, Haltiwanger, Jarmin, and Miranda, 2016). This level is also close to the firm exit rate of 9% reported by SAIC in China.

Table 7: Exogenously Chosen Parameters

| $\sigma$ | $\mu_\varphi$ | $a_h$ | $f_1 = f_2$ | $f_0$ | $\delta$ | $\alpha$ | $L$ | $\frac{H_{US}}{L+H_{US}}$ | $\frac{H_{CN}}{L+H_{CN}}$ |
|---|---|---|---|---|---|---|---|---|---|
| 4.5 | 100 | 1 | 100 | 0.1 | 8% | 0.2 | $10^8$ | 54% | 19% |

For labor-related parameters, we pin down the return to skilled labor $\alpha = 0.2$ to match the fraction of wage bill for skill-intensive occupations, i.e., managers, R&D researchers, and professionals, in the Canadian Labor Force Survey 2012. The total unskilled labor supply $L = 10^8$, which is another scaling number that only affects the measure of firms and the aggregate price. Finally, we set the relative supply of skilled worker $H_{US}$ and $H_{CN}$ to match that among populations with age 25 and above, 54% in the U.S. (World Bank 2005 data) and 19% in China (Population Census 2005) have tertiary schooling.

**Distortion-Free Parameters** We view the U.S. as a distortion-free economy. We thus calibrate 4 parameters, namely the probability of a high appeal shock $p$, the level of low appeal shock $a_l$, the standard deviation of productivity $\sigma_\varphi$ and the fixed cost of entry $f_e$, jointly to match a set of moments in the U.S. census data.

In particular, when the probability $p$ of drawing high appeal shock $a_h$ increases, the mass of firms profitable enough to produce multiple products increases. So we calibrate $p = 0.52$ to match 39% of firms are multi-product producers. Meanwhile, the level of low appeal shock $a_l$ mainly affects the relative size of the most populous measure of firms with the appeal shocks $(a_h, a_l)$. Hence, we calibrate it to be 0.45 to match the output share of MP firms. The targeted fraction and output share of MP firms are from Bernard et al. (2010).

Table 8: Calibrated Parameters

| Parameter | Value | Moment | Data | Model |
|---|---|---|---|---|
| | | Distortion-free Economy (U.S Data) | | |
| $p$ | 0.52 | MP Fraction | 0.39 | 0.39 |
| $a_l$ | 0.45 | MP Output Share | 0.87 | 0.86 |
| $\sigma_\varphi$ | 32 | Top 5% Output Share | 0.86 | 0.73 |
| $f_e$ | $2.50\times10^6$ | Entry Rate | 0.11 | 0.12 |
| | | Distorted Economy (Chinese Data) | | |
| $\xi$ | 0.80 | MP Fraction | 0.27 | 0.27 |
| $\tau_t$ | 0.42 | MP Output Share | 0.44 | 0.44 |
| $Prob(\varphi > \varphi_{tax})$ | 0.84 | Top 5% Output Share | 0.61 | 0.61 |
| $\tau_s$ | -0.20 | Subsidized Share | 0.16 | 0.16 |

*Note:* Moments with respect to the MP firms for the U.S. economy are from Bernard et al. (2010). The entry rate is from Decker et al. (2016). Moments for the Chinese economy are authors' own calculations using the ASIF data.

The standard deviation of productivity $\sigma_\varphi$ mostly influences the size of the largest firms on the right tail, and hence is calibrated to match the output share of the top

5% largest firms, 0.86, according to the Statistics of Business 2014. The fixed cost of entry $f_e$ determines the relative mass of entrants compared to incumbents. We thus calibrate $f_e$ to match the entry rate of 11% in the U.S. (Decker et al., 2016). In the equilibrium of the model, the total entry cost paid each period is $M_e L_e$ and amounts to 5.35% of unskilled labor. The upper panel of Table 8 suggests that our model closely matches the U.S. data.

**Distortion Parameters** In the final step, we calibrate the span of control parameter $\xi$ and distortion-related parameters to match moments in the Chinese ASIF data.

We first calibrate $\xi$ to be 0.8 to match the fact that MP firms are fewer in China than in the distortion-free economy. When $\xi$ decreases, operating a second product line becomes less profitable, and hence decreases the MP share. One could view the U.S. case as a special case that $\xi = 1$.

For distortions, we set $\varphi_{tax} = \varphi_{sub}$ in our calibration for simplicity. As a result, every active firm is either subsidized or taxed. Given the level of the span of control, a higher tax rate $\tau_t$ dramatically depresses the multi-product output share. We thus calibrate it to be 0.42 to match the fact that MP firms produce 44% of output. At the same time, the location of $\varphi_{tax}$ reflects the degree of size-dependent distortions, and thus mainly affects the top 5% firm's output share. Matching to the 61% output share thus gives us the level of $\varphi_{tax}$ such that $Prob(\varphi > \varphi_{tax}) = 0.84$ for an entrant. Lastly, we choose the subsidy rate to be -0.20 to match the equilibrium share of subsidized firms (negative log TFPR), 0.16, in the ASIF data. The bottom panel of Table 8 suggests that combined with previously calibrated parameters, our model also well matches the Chinese data.

## 4.2 Roles of $\tau$, $\xi$ and $\alpha$

We next discuss how targeted moments change when we vary distortions $\tau$, the span of control parameter $\xi$, and the return to skilled labor $\alpha$ to shed light on model mechanisms and the source of identifications.

Let the welfare $Q$ in the distortion-free China case be 100%. The top panel of Table 9 suggests that the welfare in the benchmark model with distortions is only 52.3%.

**Role of $\tau$** When both tax and subsidy distortions are removed, the fraction of MP firms barely changes, while their output share, as well as the output share of the top 5% largest firms, increase to 51.21% and 72.39% in the panel *Role of* $\tau$. The relative size of an MP firm thus increases from 2.12 to 2.75. More firms come in with a higher expected profit, and the welfare level is 100%.

Imposing tax or subsidy distortions alone cannot generate moments close to the data. When we keep the tax distortions alone, the targeted moments except for the subsidized firm share are almost the same as those in the distortion-free case. A similar observation can be found when we keep the subsidy distortions alone. This result suggests that to match output shares of MP and top 5% firms in the data, we need to have tax distortions to scale down most productive firms, and subsidy distortions to scale up least productive ones simultaneously. Meanwhile, the measure of firms and welfare are both lower in the *Tax only* economy and higher in the *Subsidy only* economy.

**Role of $\xi$** We choose two alternative levels, 0.9 and 0.4, for $\xi$ to illustrate its impact on model moments.

An increase in $\xi$ increases the profitability of the second product, and hence considerably increases the fraction and the output share of MP firms. For the top 5% largest firms, their output share is, however, not changed much by changes in $\xi$ when we fix $\tau$. Similarly for the measure of firms and the welfare. These measures are more affected by the existence of distortions. For the relative size of MP firms, there seems no monotonic effect coming from $\xi$.

**Role of $\alpha$** We also tune the return to skilled labor parameter $\alpha$ into two alternative levels, 0.24 and 0.16.

When $\alpha$ increases, the return to skilled labor in the appeal production increases. Productive firms employ more skilled labor and the equilibrium wage $w_h$ increases. This lowers the profit margin, causes some marginal firms to drop the second product and some others to stop production. Therefore, the number and output shares of MP firms decrease, while the output share of the top 5% firms increases. With a lower expected profit, fewer firms enter, and the measure of firms declines. However, the welfare level increases considerably due to the increased per-product appeal by the reallocation of skilled labor to more productive firms. Again, we do not see a monotonic change in the relative size of an MP firm when $\alpha$ changes.

Table 9: Changes of Moments by Varying $\tau$, $\xi$ and $\alpha$

| | Fraction (Multi) | Output Share (Multi) | Top 5% Output Share | Subsidized Firm Share | MP/SP Size | Measure of Firms | Welfare |
|---|---|---|---|---|---|---|---|
| Data | 0.2700 | 0.4400 | 0.6100 | 0.1600 | 2.12 | - | - |
| Model | 0.2732 | 0.4423 | 0.6104 | 0.1627 | 2.11 | 200.67 | 52.30% |
| | | | Role of $\tau$ | | | | |
| Distortion-free | 0.2763 | 0.5121 | 0.7239 | 0.0000 | 2.75 | 251.27 | 100% |
| Tax Only | 0.2802 | 0.5046 | 0.7361 | 0.0000 | 2.62 | 159.98 | 51.14% |
| Subsidy Only | 0.2525 | 0.5120 | 0.7411 | 0.0861 | 3.11 | 274.91 | 100% |
| | | | Role of $\xi$ | | | | |
| $\xi = 0.9, \tau = 0$ | 0.3313 | 0.7051 | 0.7235 | 0.0000 | 4.83 | 251.82 | 100.69% |
| $\xi = 0.9, \tau \neq 0$ | 0.3223 | 0.5970 | 0.6120 | 0.1628 | 3.11 | 193.51 | 52.47% |
| $\xi = 0.4, \tau = 0$ | 0.0044 | 0.1621 | 0.7241 | 0.0000 | 43.15 | 253.03 | 99.83% |
| $\xi = 0.4, \tau \neq 0$ | 0.0021 | 0.0999 | 0.6123 | 0.1600 | 51.60 | 198.64 | 52.12% |
| | | | Role of $\alpha$ | | | | |
| $\alpha = 0.24, \tau = 0$ | 0.1089 | 0.3524 | 0.9507 | 0.0000 | 4.45 | 61.58 | 202.23% |
| $\alpha = 0.24, \tau \neq 0$ | 0.1393 | 0.3523 | 0.9048 | 0.2493 | 3.36 | 44.71 | 103.64% |
| $\alpha = 0.16, \tau = 0$ | 0.4437 | 0.8859 | 0.5396 | 0.0000 | 9.74 | 457.78 | 54.48% |
| $\alpha = 0.16, \tau \neq 0$ | 0.4792 | 0.8661 | 0.4423 | 0.1660 | 7.03 | 316.09 | 28.72% |

*Note:* $\tau = 0$ indicates the removal of distortions, while $\tau \neq 0$ means that the experiment has the same distortions as in the benchmark model.

## 4.3 Distorted Cutoff Productivity Levels

The number of products could change when a firm is imposed by a distortion $\tau$ compared to the case that it is not. This may alter the productivity compositions of firms that produce one product and two products.

**Changes in Cutoffs** To illustrate the distorted product margin, Table 10 lists cutoff productivities in percentiles for firms who receive different appeal shocks in the two economies, i.e., distortion-free China and distorted China. The former has the same parameterization as the latter except for the removal of distortions.

In the economy of distortion-free China, 82.95% of firms enter and produce at least one product, conditional on drawing appeal shocks $(a_h, a_h)$. Among them, about 79%, produce 2 products. For firms that receive appeal shocks $(a_h, a_l)$, the cutoff productivity of entry is the same as that for firms with $(a_h, a_h)$ shocks as the model shows in Section 3. Yet the lower appeal shock for the potential second product-line causes a much smaller fraction of firms to produce 2 products, 0.22% conditional on receiving $(a_h, a_l)$. Finally for firms who receive shocks $(a_l, a_l)$, 4.87% of them enter and 0.22% produce 2 products. Note that the cutoff for producing 2 products is also the same as firms with appeal shocks $(a_h, a_l)$.

In the economy of distorted China, firms are taxed if the productivity $\varphi$ is higher than the 84% percentile, and subsidized if it is below. Cutoff changes for firms with tax-like distortions are as follows. With appeal shocks $(a_h, a_h)$, 80.57% of them enter and 61.67% produce 2 products, compared to 82.95% and 65.79% absent distortions. In other words, about 2.38% of entrants inefficiently choose not to enter, and about 4.12% of them inefficiently reduce the number of products from 2 to 1. These numbers are 2.38% and 0.12% for firms with appeal shocks $(a_h, a_l)$, and 1.68% and 0.12% for firms with appeal shocks $(a_l, a_l)$.

For firms with subsidy distortions, cutoffs change in the opposite direction. With appeal shocks $(a_h, a_h)$, the cutoff percentile for firms to enter is 98.73%, and 97.79% to produce 2 products. This suggests that the mass of firms in the productivity range of 84 percentile to 98.73 percentile, i.e., 14.73%, inefficiently enter. And firms in the range of 84 percentile to 97.79 percentile, i.e., 13.79% of firms, inefficiently produce 2 products. Similarly, 14.73% of firms with appeal shocks $(a_h, a_l)$ and 3.86% for firms with appeal shocks $(a_l, a_l)$ inefficiently enter. We find no subsidized firms producing 2 products with the latter two appeal types.

Table 10: Cutoff Productivity in Percentiles, Distorted vs Distortion-Free China

| | Type of Appeal Shocks | | |
|---|---|---|---|
| | $(a_h, a_h)$ | $(a_h, a_l)$ | $(a_l, a_l)$ |
| Distortion-free China | | | |
| *1+ Product* | 82.95 | 82.95 | 4.87 |
| *2 Products* | 65.79 | 0.22 | 0.22 |
| Distorted China | | | |
| − Taxed | | | |
| *1+ Product* | 80.57 | 80.57 | 3.19 |
| *2 Products* | 61.67 | 0.10 | 0.10 |
| − Subsidized | | | |
| *1+ Product* | 98.73 | 98.73 | 87.86 |
| *2 Products* | 97.79 | 84.00 | 84.00 |

*Note:* The lower the percentile, the higher the cutoff productivity is. The block of ”− Taxed” lists cutoff productivities for firms that receive a tax distortion. Vice versa for the block of ”− Subisdized”. The row of 1+ product lists the cutoff productivities for firms that enter and produce 1 or more products in equilibrium, similarly defined for the row of 2 products.

**Changes in Mass of Firm Types** Analysis above shows how distortions change the location of productivity cutoffs. These changes have to be combined with the stochastic process of appeal shocks and the distribution of productivity $\varphi$ in order to quantify the effect of distortions. We thus compute the fractions and output shares of firms with different appeal shocks and choices in the number of products. Table 11 compares changes in these shares across three economies, i.e., the U.S., the distortion-free China, and the distorted China.

In the distorted-free China case, multi-product firms, i.e., 2 product producers, are mostly from firms with appeal shocks $(a_h, a_h)$ (27.38% out of 27.63%). Very few firms with appeal shocks $(a_h, a_l)$ and $(a_l, a_l)$ produce 2 products. These MP firms produce 51.21% of the aggregate output.

In the distorted China case, the fraction of MP firms is 27.32%, only 0.31% lower than in the distortion-free case. However, the negligible drop in their fraction masks the composition change of MP firms: an increased mass of unproductive and subsidized MP firms at the cost of productive and taxed ones. Specifically, subsidized firms that receive $(a_h, a_h)$, which should have not entered absent distortions, have an equilibrium mass of 4.96% producing 2 products and 8.01% of aggregate output. Meanwhile, among MP firms that receive tax distortions, the mass with appeal shocks $(a_h, a_h)$ decreases from 27.38% to 22.23%, with a declining output share from 36.77% to 28.73%. Similarly, the mass of MP firms with appeal shocks $(a_h, a_l)$ decreases from 0.08% to 0.03%, with a declining output share from 14.44% to 7.47%. Therefore, Table 11 echoes our finding of the distorted product margin in the empirical analysis.

**Fewer and Smaller** Table 11 reveals sources of why MP firms are fewer and smaller in China compared to in the U.S. According to our model setup, the two economies differ in distortions $\tau$, the span of control parameter $\xi$, and the skilled worker intensity $\frac{H}{H+L}$.

The difference in the fractions and output shares of MP firms between distortion-free China and the U.S. comes from the span of control parameter $\xi$. It is not affected by the skilled worker intensity $\frac{H}{H+L}$, because the intensity does not alter productivity cutoffs. With a higher $\xi$ in the U.S. economy, more firms find it profitable to produce 2 products. In particular, 34.44% of firms produce 2 products given the $(a_h, a_h)$ shocks with an output share of 51.98%. These numbers are 1.87% and 36.85% for firms with appeal shocks $(a_h, a_l)$, 4.06% and a negligible share for those with shocks $(a_l, a_l)$.

We thus conclude that the fewer and smaller facts of MP firms in China are primarily explained by the span of control parameter $\xi$. When $\xi$ drops to 0.8, the fraction of MP firms decreases from 39% to 27.63%, and the relative size from 10.46 to 2.75. Adding distortions further pushes the fraction to 27.32% and the relative size to 2.14. Quantitatively, distortions account for 3% and 7% of the gaps of the fraction difference and the average size difference between China and the U.S., respectively. This result indicates the importance of studies on the cross-country difference in management practices (Bloom and Van Reenen, 2007; Bloom et al., 2020).

Table 11: Firm Type, Fraction and Output Share in U.S., Distortion-Free China and Distorted China

| | Firm Type | | | | | |
|---|---|---|---|---|---|---|
| | $(a_h, a_h; 1)$ | $(a_h, a_h; 2)$ | $(a_h, a_l; 1)$ | $(a_h, a_l; 2)$ | $(a_l, a_l; 1)$ | $(a_l, a_l; 2)$ |
| | U.S. ($\xi = 1, \frac{H}{H+L} = 54\%$) | | | | | |
| Fraction | 0.04% | 34.44% | 59.59% | 4.06% | 0.00% | 1.87% |
| Output Share | 0.00% | 51.98% | 11.16% | 36.85% | 0.00% | 0.00% |
| | Distortion-Free China ($\xi = 0.80, \frac{H}{H+L} = 19\%$) | | | | | |
| Fraction | 7.14% | 27.38% | 63.58% | 0.17% | 1.65% | 0.08% |
| Output Share | 0.01% | 36.77% | 48.78% | 14.44% | 0.00% | 0.00% |
| | Distorted China ($\xi = 0.80, \frac{H}{H+L} = 19\%$) | | | | | |
| − Taxed Firms | | | | | | |
| Fraction | 6.81% | 22.23% | 53.55% | 0.06% | 0.95% | 0.03% |
| Output Share | 0.02% | 28.73% | 41.95% | 7.47% | 0.00% | 0.00% |
| − Subsidized Firms | | | | | | |
| Fraction | 0.34% | 4.96% | 9.79% | 0.00% | 1.18% | 0.00% |
| Output Share | 0.00% | 8.01% | 13.78% | 0.00% | 0.00% | 0.00% |

*Note:* Firm type $(a_i, a_j; n)$ represents the group of firms who receive appeal shocks $(a_i, a_j)$, $i, j \in \{h, l\}$ and produce $n$ products in equilibrium.

## 4.4 Firm and Product Margins of Welfare Loss

The welfare level of our benchmark model in Table 9 indicates a loss of 47.7%, compared to the distortion-free economy of China. This section decomposes this loss into three margins: output intensive, firm extensive, and product extensive, to understand the quantitative importance of our novel product margin. To do so, we implement three counterfactual experiments. In all experiments, distortions are removed and labor markets clear. Their difference lies in whether the cutoffs of product choices or the measure of firms is preserved as in the benchmark model.

**Experiments** Our first experiment keeps the cutoffs of product choices and the measure of firms unchanged as in the benchmark model, i.e., Product-Fixed and Firm-Fixed (PF-FF). In this experiment, firms reset product prices and quantities to maximize their variable profits, taking the new aggregate prices given. Variable profits are revenues less variable labor costs. Thus, firms subsidized in the benchmark model may have a positive variable profit and a negative profit but are kept producing by the idea of preserving the cutoffs of product choices. For entry and exit, we replace exiting firms by the same mass of firms that enter under the entry rule in the benchmark model.

Our second experiment relaxes the product margin by letting firms to re-optimize their product decisions while keeping the measure of firms unchanged as in the benchmark model. We name this as the Firm-Fixed (FF) experiment. In this experiment, productive firms add products while those previously subsidized ones may drop products and even stop producing. We keep those non-producing subsidized firms as part of the measure of firms and replace exits by entrants as in the PF-FF experiment.

The third experiment relaxes the firm margin by allowing firms to freely enter while keeping cutoffs of product choices unchanged as in the benchmark model. We name this as Product-Fixed (PF) experiment. In this experiment, new firms enter because of an increased expected profit by the removal of distortions.

Table 12: Decomposition of Misallocation

| | Benchmark | PF-FF | FF | PF | Distortion-Free |
|---|---|---|---|---|---|
| Measure of Firms | 200.67 | 200.67 | 200.67 | 210.30 | 251.27 |
| Relative Top 5% quantity | 44.09% | 67.59% | 98.80% | 100% | 100% |
| Relative Variety | 79.67% | 79.67% | 66.84% | 83.49% | 100% |
| Skilled Wage | 0.5338 | 0.5117 | 0.8002 | 0.7853 | 0.7853 |
| Aggregate Price | 2.22 | 1.31 | 1.31 | 1.26 | 1.22 |
| Welfare | 52.30 % | 60.71% | 95.24% | 97.28% | 100% |

*Note:* Calibrated Chinese economy with distortions. PF-FF: Distortions are removed while both the product cutoffs, the distribution of firm, and the firm measure are fixed as in the Benchmark. PF: Distortions are removed while the product cutoffs are fixed. And the measure of firms is adjusted. FF: Distortions are removed, firms are free to choose product but the distribution of firms and its measure are fixed.

Table 13: Cutoff Productivity in Percentiles in the FF Experiment

| | Type of Appeal Shocks | | |
|---|---|---|---|
| | $(a_h, a_h)$ | $(a_h, a_l)$ | $(a_l, a_l)$ |
| FF Experiment | | | |
| *1+ Product* | 86.47 | 86.47 | 9.27 |
| *2 Products* | 72.54 | 0.74 | 0.74 |

*Note:* The lower the percentile, the higher the cutoff productivity is. The row of 1+ product lists the cutoff productivities for firms that enter and produce 1 or more products in equilibrium, similarly defined for the row of 2 products.

**Results** Table 12 presents welfare levels for all experiments along with several key statistics. In the PF-FF experiment, the welfare level is 60.71%, which is 8 percentage points higher than the benchmark model. We define such an increase as the *output intensive margin*, which is conceptually close to the static misallocation measure of Hsieh and Klenow (2009). In their paper, welfare improves when resources are reallocated among existing firms to equalize revenue productivities (TFPR). Our PF-FF experiment replicates such an equalization at a finer level, i.e., across firms and across products.

When we move from the PF-FF economy to the FF economy, an additional 34.54 percentage point increase in welfare (95.24%-60.71%) occurs if firms re-optimize product choices, prices, and quantities. We call this increase of welfare as the *product extensive margin* that is caused by three economic forces.

First, unproductive and previously subsidized firms drop products. For instance, Table 13 shows that with appeal shocks $(a_h, a_h)$, firms from the top 98.73% to 86.74% and those from the top 97.79% to 84% switch to producing 0 product (from 1) and 1 product (from 2), respectively. Second, productive and previously taxed firms add products. With appeal shocks $(a_h, a_l)$ or $(a_l, a_l)$, only 0.1% productive firms with a tax-like distortion produce 2 products in the PF-FF economy, while the percentile increases to a top 0.74% in the FF economy. Third, expansions in the number of products on the right tail of the distribution increase the aggregate labor demand and wages shown in Table 12. In the equilibrium of the FF economy, this narrows the profit margin for firms with lower $\varphi$s, and induces the reallocation of labor towards firms with higher $\varphi$s. Hence, the welfare further increases by more output with better appeals.

When we compare the PF economy to the PF-FF economy, there is a similar increase in welfare, 36.57 percentage points (97.28% - 60.71%), when firms are allowed to enter freely. In this economy, the product decisions follow cutoff rules in the benchmark model in Table 10. We define this increase of welfare as the *firm extensive margin*. The gain is induced by an increase in the measure of firms since the removal of distortions increases the expected profit of entry.

**Discussions** Results above show the quantitative closeness of the product and firm extensive margins. One may wonder whether the result is caused by our setup of the utility function. From the household perspective, a new product added by an incumbent firm is equivalent to that produced by a new single-product firm. However, the productivity level of incumbent firms that add products in the FF economy could differ from that of a new single-product firm in the PF economy, as indicated in our discussion of cutoff changes. Thus, there is not necessarily a similar gain in both experiments.

The comparable magnitude in the two margins is also consistent with the equal importance of product add-and-drop and firm entry-and-exit in the U.S. manufacturing production (Bernard et al., 2010). Their paper shows how empirically the reallocation through two margins happens simultaneously, while our model provides a way to shut down one margin to highlight the other.

One may conjecture another significant increase of welfare from the PF or FF economy to the distortion-free economy since such a change relaxes the other margin. We do not see this phenomenon. In other words, the welfare levels in the FF economy and the PF economy are each close to that in the distortion-free case. This is due to the equilibrium force of increased wages interacted with the granularity of the firm size distribution. When firms re-optimize productions in the FF economy, resource reallocation towards productive firms on the right tail of the distribution, which is granular in our calibration with $\alpha = 0.2$. A similar mechanism happens when new firms enter in the PF economy. This mechanism is best illustrated by the measure of top 5% quantity share, which computes the appeal adjusted quantity share of top 5% firms in the counterfactual economy compared to the one in the distortion-free economy. Table 12 shows that the measures are close to 100% in both PF and FF economies.

## 4.5 Extension of Continuous Distortions

This extended section compares model moments if we alternatively set the distortion continuous in the firm-level productivity $\varphi$ as in Jaef (2018). The magnitude of misallocation in our benchmark economy turns out robust to this alternative distortion setting.

Table 14: Changes of Moments when Distortion is Continuous

| | Fraction (Multi) | Output Share (Multi) | Top 5% Output Share | Subsidized Firm Share | MP/SP Size | Measure of Firms | Welfare |
|---|---|---|---|---|---|---|---|
| Data | 0.2700 | 0.4400 | 0.6100 | 0.1600 | 2.12 | - | - |
| $\gamma$=0 | 0.2763 | 0.5121 | 0.7239 | 0.0000 | 2.75 | 157.90 | 100% |
| $\gamma$=0.1 | 0.2906 | 0.4517 | 0.6554 | 0.0000 | 2.01 | 151.99 | 53.17% |
| $\gamma$=0.25 | 0.3134 | 0.3896 | 0.5293 | 0.0000 | 1.40 | 68.49 | 21.85% |
| $\gamma$=0.5 | 0.3414 | 0.3678 | 0.2852 | 0.0000 | 1.12 | 18.40 | 5.38% |

Following the literature, we specify the continuous size-dependent distortion as

$$1 - \tau(\varphi) = \varphi^{-\gamma} \tag{32}$$

where $\gamma \in (0, 1)$, indicating a monotonically increasing $\tau$ in $\varphi$. For firms with productivity $\varphi > 1$, $\tau(\varphi) > 0$. For firms with productivity $\varphi < 1$, $\tau(\varphi) < 0$.

Given this distortion function, Table 14 reports the associated equilibrium outcomes for four levels of $\gamma$, 0, 0.1, 0.25 and 0.5 when $\xi = 0.8$ (see appendix for model solutions). The case of $\gamma = 0$ corresponds to the distortion-free economy in previous tables. When $\gamma$ equals 0.25, i.e., the degree of size-dependence distortions for China according to Jaef (2018), there are too many small MP firms. Their number and output shares are 31.14% and 38.96%. Similarly, the top 5% largest firms are too small, taking up only 52.93% of the aggregate output. This is unsurprising since more productive firms are taxed more than in our benchmark economy.

If we perturb $\gamma$ to several alternative values, e.g., 0.5 and 0.1, we find that the case of $\gamma = 0.1$ best matches the targeted moments except for the fraction of firms who are subsidized in the data. Here the zero fraction of subsidized firms is a result of our normal assumption of $\varphi$ with a mean value of 100. The welfare level for this economy is 53.17%, which is fairly close to the level of 52.30% in our benchmark model. Hence, the magnitude of misallocation is robust to the specification of distortions in this multi-product economy under our parameterization.

# 5 Conclusion

This paper empirically studies the distorted product margin, and quantifies the magnitude it accounts for misallocation in a general equilibrium model of discrete product choices with firm entry and exit.

The empirical part establishes several facts on multi-product firms in a developing economy like China. First, multi-product firms are fewer and smaller in China, compared to their counterparts in the U.S. Second, the multi-product probability of a firm decreases with an increased level of distortion. This result holds both when we use the indirect measure of distortion, i.e., log TFPR, and when we use a direct measure, i.e., corporate income tax change as well as firm self-reported distortions.

We then build the discrete product choices into a canonical model of firm entry and exit à la Melitz (2003), and impose size-dependent distortions on firms with heterogeneous productivities. Consistent with the data, the equilibrium cutoff for producing multiple products increases for firms that are taxed, and vice versa for those that are subsidized. Under our calibration, the welfare level in the distorted benchmark model is 52.30%, compared to the distortion-free economy of China. Further counterfactual experiments suggest a sizable welfare increase (34.53 percentage points) from eliminating the distorted product margin. This is quantitatively close to the extensive margin of firm entry and exit (36.57 percentage points). The complementarity between the product and firm extensive margins only increases the welfare by 2.72 to 4.76 percentage points, due to the general equilibrium forces interacted with the granularity of largest firms.

Our paper highlights the importance of distorted product margins in shaping misallocation. It provides a novel perspective to understand the phenomenon of missing large firms in developing economies (Hsieh and Olken, 2014; Hsieh and Klenow, 2014; Bento and Restuccia, 2020). It also suggests that in terms of long-run welfare gains, policies that remove barriers for product innovations is as important as those that encourage firm entries.

# References

Ackerberg, Daniel A., Kevin Caves, and Garth Frazer (2015), "Identification Properties of Recent Production Function Estimators." *Econometrica*, 3, 2411–2451.

Alfaro, Laura and Anusha Chari (2014), "Deregulation, misallocation, and size: Evidence from india." *The Journal of Law and Economics*, 57, 897–936.

Argente, David, Munseob Lee, and Sara Moreira (2019), "The life cycle of products: Evidence and implications." *Available at SSRN 3163195*.

Bai, Yan, Dan Lu, and Xu Tian (2016), "Do Financial Frictions Explain Chinese Firms' Saving and Misallocation?" *2016 Meeting Papers. No. 998. Society for Economic Dynamics, 2016*.

Barseghayn, Levon and Riccardo DiCecio (2011), "Entry Costs, Industry Structure, and Cross-country Income and TFP Differences." *Journal of Economic Theory*, 146(5), 1828–1851.

Bento, Pedro and Diego Restuccia (2020), "On average establishment size across sectors and countries." *Journal of Monetary Economics*.

Bernard, Andrew B., Stephen J. Redding, and Peter K. Schott (2010), "Multi-Product Firms and Product Switching." *American Economic Review*, 100(1), 70–97.

Bernard, Andrew B, Stephen J Redding, and Peter K Schott (2011), "Multiproduct firms and trade liberalization." *The Quarterly journal of economics*, 126, 1271–1318.

Bloom, Nicholas, Kalina Manova, John Van Reenen, Stephen Teng Sun, and Zhihong Yu (2020), "Trade and management." *Review of Economics and Statistics*, 1–44.

Bloom, Nicholas and John Van Reenen (2007), "Measuring and explaining management practices across firms and countries." *The quarterly journal of Economics*, 122, 1351–1408.

Boedo, Hernan J Moscoso and Aslı Şenkal (2014), "Misallocation, informality, and human capital: Understanding the role of institutions." *Journal of Economic Dynamics and Control*, 42, 122–142.

Brandt, Loren, Johannes Van Biesebrock, and Zhang (2014), "Challenges of Working with the Chinese NBS Firm-level data." *China Economic Review*, 30, 339–352.

Broda, Christian and David E. Weistein (2010), "Product Creation and Destruction: Evidence and Price Implications."

Caliendo, Lorenzo and Esteban Rossi-Hansberg (2012), "The impact of trade on organization and productivity." *The quarterly journal of economics*, 127, 1393–1467.

Chen, Qiaoyi, Zhao Chen, Zhikuo Liu, Juan Carlos Suárez Serrato, and Daniel Yi Xu (2021), "Regulating conglomerates: Evidence from an energy conservation program in china."

Clementi, Gian Luca and Berardino Palazzo (2016), "Entry, Exit, Firm Dynamics and Aggregate Fluctuations." *American Economic Journal: Macroeconomics*, 8(3), 1–41.

De Loecker, Jan and Jan Eeckhout (2018), "Global market power." Technical report, National Bureau of Economic Research.

De Loecker, Jan and Frederic Warzynski (2012), "Markups and Firm-Level Export Status." *American Economic Review*, 102(6), 2437–2471.

Decker, Ryan A, John Haltiwanger, Ron S Jarmin, and Javier Miranda (2016), "Declining business dynamism: What we know and the way forward." *American Economic Review*, 106, 203–07.

Eckel, Carsten and J. Peter Neary (2010), "Multi-Product Firms and Flexible Manufacturing in the Global Economy." *Review of Economic Studies*, 77, 188–217.

Garcia-Santana, Manuel and Josep Pijoan-Mas (2014), "The reservation laws in india and the misallocation of production factors." *Journal of monetary economics*, 66, 193–209.

Gibbons, Robert S and John Roberts (2013), *The handbook of organizational economics*. Princeton University Press.

Goldberg, Pinelopi K, Amit K Khandelwal, Nina Pavcnik, and Petia Topalova (2010), "Multiproduct firms and product turnover in the developing world: Evidence from india." *The Review of Economics and Statistics*, 92, 1042–1049.

Gourio, François and Nicolas Roys (2014), "Size-dependent regulations, firm size distribution, and reallocation." *Quantitative Economics*, 5, 377–416.

Gourio, Francois and Leena Rudanko (2014), "Customer capital." *Review of Economic Studies*, 81, 1102–1136.

Guner, Nezih, Gustavo Ventura, and Yi Xu (2008), "Macroeconomic implications of size-dependent policies." *Review of Economic Dynamics*, 11, 721–744.

Hottman, Colin J., Stephen J. Redding, and David E. Weistein (2016), "Quantifying the Sources of Firm Heterogeneity." *Quarterly Journal of Economics*.

Hsieh, Chang-Tai and Peter J. Klenow (2009), "Misallocation and Manufacturing TFP in China and India." *The Quarterly Journal of Economics*, 124 (4), 1403–1448.

Hsieh, Chang-Tai and Peter J. Klenow (2014), "The Life Cycle of Plants in India and Mexico." *The Quarterly Journal of Economics*, 129(3), 1035–1084.

Hsieh, Chang-Tai and Benjamin A. Olken (2014), "The Missing "Missing Middle"." *Journal of Economic Perspectives*, 28(3), 89–108.

Jaef, Roberto N Fattal (2018), "Entry and exit, multiproduct firms, and allocative distortions." *American Economic Journal: Macroeconomics*, 10, 86–112.

Kehrig, Matthias and Nicolas Vincent (2019), "Good dispersion, bad dispersion." Technical report, National Bureau of Economic Research.

Klette, Tor Jakob and Samuel Kortum (2004), "Innovating firms and aggregate innovation." *Journal of political economy*, 112, 986–1018.

Li, Xiaoxue, Liu Tian, and Jing Xu (2020), "Missing social security contributions: the role of contribution rate and corporate income tax rate." *International Tax and Public Finance*, 27, 1453–1484.

Manova, Kalina and Zhihong Yu (2017), "Multi-product firms and product quality." *Journal of International Economics*, 109, 116–137.

Mayer, Thierry, Marc J Melitz, and Gianmarco IP Ottaviano (2014), "Market size, competition, and the product mix of exporters." *American Economic Review*, 104, 495–536.

Melitz, Marc (2003), "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica*, 71, 1695–1725.

Midrigan, Virgiliu and Daniel Yi Xu (2014), "Finance and Misallocation: Evidence from Plant-Level Data." *American Economic Review*, 104(2), 422–458.

Minniti, Antonio and Francesco Turino (2013), "Multi-product firms and business cycle dynamics." *European Economic Review*, 57, 75–97.

Nocke, Volker and Stephen Yeaple (2014), “Globalization And Multiproduct Firms.” *International Economic Review*, 55, 993–1018.

Poschke, Markus (2018), “The firm size distribution across countries and skill-biased change in entrepreneurial technology.” *American Economic Journal: Macroeconomics*, 10, 1–41.

Restuccia, Diego (2014), “Barriers to capital accumulation in a model of technology adoption and schooling.” *Journal of Economic Dynamics and Control*, 44, 81–91.

Restuccia, Diego and Richard Rogerson (2008), “Policy Distortions and Aggregate Productivity with Heterogeneous Establishments.” *Review of Economic Dynamics*, 11(4), 707–720.

Restuccia, Diego and Richard Rogerson (2013), “Misallocation and Productivity.” *Review of Economic Dynamics*, 16, 1–10.

Ševčík, Pavel (2017), “Do multi-plant firms reduce misallocation? evidence from canadian manufacturing.” *The Journal of Industrial Economics*, 65, 275–308.

Stokey, Nancy L (1991), “Human capital, product quality, and growth.” *The Quarterly Journal of Economics*, 106, 587–616.

# Appendix

## A Data

**Corporate Income Tax Reductions** Changes in corporate income tax rates are illustrated in Figure A1 from Li, Tian, and Xu (2020).

In the 2004-2012 (excluding 2010) balanced panel, we first create a dummy variable, $Treat_t$, which equals 0 if the firm is located in western China, or is in high-tech industries according to Table A1 or has their address located in economic zones, i.e., *kaifaqu* in Chinese. The $Treat_t$ dummy equals 1 otherwise. Firms in this panel had sales above 20 million yuan and did not satisfy the small enterprise criterion.

To fix the treatment and control groups before 2008, we define the treatment group that has $Treat_t$ all equal to 1 from 2004 to 2007. There are observations that change the group type across years by measurement errors. The correlation between $Treat_t$ and $Treat_s$ in any two years $t$ and $s$ is above 0.91.

Similarly we define the $Exporter$ dummy. For each year, we define an $Exporter_t$ dummy and set $Exporter$ 1 if firms exported every year from 2004 to 2007. The exporting status of firms showed a smaller persistence, with a correlation coefficient of 0.58 between 2004 and 2007. We also try an alternative definition of $Exporter$, which equals 1 if firms export in any year from 2004 to 2007. The difference-in-difference estimation gives results similar to Table 4.

**WBES Data Questions** The distortion measures come from the following WBES 2012 questions exampled in Figure A1. Questions D.30, E.30, G.30, I.30, K.30, J.30 and L.30 are on other aspects of firm operations as in Table 5, and similarly formatted.

The multi-product dummy variable comes from the question shown in Figure A3. Firms are asked the percentage of sales generated by the main product in this question and coded into the variable d1a3, of which the distribution is plotted in

Figure A1: The Corporate Income Tax Reform in 2008

| | Pre-tax reform | | Post-tax reform |
|---|---|---|---|
| | Domestic | Foreign | All |
| Baseline tax rate | 33 | 33 | 25 |
| Reduced tax rate | | | |
| West Development program | 15 | 15 | 15 |
| Small enterprises[a] | 18 | – | 10–20 |
| | 27 | – | |
| High-tech enterprises[b] | 15 | – | 15 |
| Economic zones[c] | – | 15 | 18–25 |
| | – | 24 | 25 |

[a]Prior to the tax reform, domestic-owned firms with taxable profit at or below RMB 30,000 were taxed at 18%; firms with taxable profit between RMB 30,001 to 100,000 were taxed at 27%. After the tax reform, a reduced tax rate of 20% applied to all small firms that fulfill certain requirements for asset level and employment (namely the small and low-profit enterprises), regardless of ownership. Starting in 2010, a reduced tax rate of 10% was introduced for small and low-profit enterprises with taxable profit at or below RMB 30,000

[b]Prior to the tax reform, domestic-owned high-tech firms operating in the High-Tech Industrial Development Zones were eligible for a 15% tax rate. After the tax reform, all high-tech firms regardless of ownership and location were eligible for a 15% tax rate

[c]Prior to the tax reform, foreign-owned firms in cities with economic zones enjoyed a reduced rate of 15% (located inside economic zones) or 24% (located outside economic zones, but inside the cities of the economic zones). After the tax reform, the reduced rate was gradually eliminated. The 15% rate was increased to 18% in 2008, 20% in 2009, 22% in 2010, 24% in 2011, and 25% in 2012. The 24% tax rate has been 25% since 2008

*Source*: The State Administration of Taxation

*Source:* Table 1 from Li et al. (2020).

Table A1: List of High-Tech Industries, 2002-2013 Standard

| CIC 2-digit code | Industry |
|---|---|
| 25 | Nuclear fuel processing |
| 26 | Information chemical manufacturing |
| 27 | Manufacture of medicines |
| 36 | Medical equipment and equipment manufacturing |
| 37 | Aerospace and aircraft manufacturing |
| 40 | Manufacture of communication equipment, computers and other electronic equipment |
| 41 | Manufacture of measuring instruments and machinery for office |

*Note:* The 2-digit Chinese Industry Classification (CIC) codes are in the 2003 standards. Definitions of high-tech industries are following the 2002-2013 standards. There are later revisions in 2013 and 2017.

Figure A2: WBES Sample Question on Distortions

| **C.30** | Using the response options on the card; To what degree is **Electricity** an obstacle to the current operations of this establishment? **SHOW CARD 3**<br><br>Using the response options on the card; To what degree is **Telecommunications** an obstacle to the current operations of this establishment? **SHOW CARD 3** |
|---|---|

| | | No obstacle | Minor obstacle | Moderate obstacle | Major obstacle | Very Severe Obstacle | **Don't Know (spontaneous)** | **Does Not Apply (spontaneous)** |
|---|---|---|---|---|---|---|---|---|
| Electricity | **c30a** | 0 | 1 | 2 | 3 | 4 | **-9** | **-7** |
| Telecommunications | **c30b** | 0 | 1 | 2 | 3 | 4 | **-9** | **-7** |

Source: World Bank Enterprise Survey China Manufacturing Module 2012

Figure A3: WBES Question on the Multi-Product Status

| | |
|---|---|
| **D.1a3** | What percentage of total sales does the main product represent? |

| | **Percent** |
|---|---|
| Percentage of sales represented by main product | **d1a3 %** |
| **Don't know (spontaneous)** | **-9** |

Source: World Bank Enterprise Survey China Manufacturing Module 2012

Figure A4. If a firm's main product exceeds 90% or alternatively 95%, we classify it as an SP firm. The reason is that the histograms in Figure A4 are cluttered in percentages of 100, 95, 90, 85, 80 and etc, while 90 and 95 give the closest fractions of MP firms compared to the numbers in the AIES data and in the census data.

# B Model

## B.1 Consumer's Problem

We derive the inverse demand from the consumer's problem.

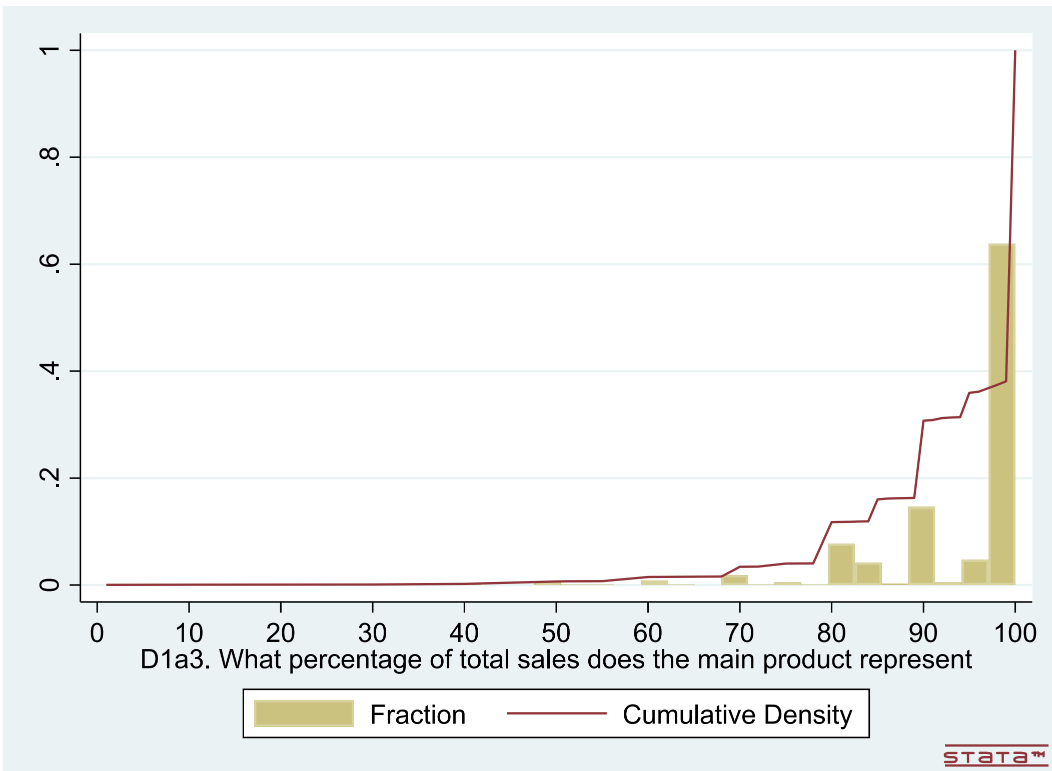
$$\min_{q(\omega)} \left\{ \int p(\omega)q(\omega)d\omega - \eta \left[ \left[ \int_{\omega\in\Omega} (\lambda(\omega)q(\omega))^{\rho} \, d\omega \right]^{\frac{1}{\rho}} - \bar{U} \right] \right\}$$

FOC for the consumption good $q(\omega)$: $\frac{p_i}{\lambda_i} = \eta U^{1-\rho} (\lambda_i q_i)^{\rho-1}$. This implies that the relative demand for two goods is $\frac{p_i}{p_j}\frac{\lambda_j}{\lambda_i} = \left(\frac{\lambda_i q_i}{\lambda_j q_j}\right)^{\rho-1}$

Let $R \equiv \int p(\omega)q(\omega)d\omega$. We have $R = \int \frac{p_i}{\lambda_i}\lambda_i q_i di = \int \eta U^{1-\rho} (\lambda_i q_i)^{\rho} \, di = \eta U$. Define $\eta \equiv P$ and $Q$. Note, the aggregate quantity $Q$ is a quality adjusted aggregate quantity. In our model, $R = \int p(\omega)q(\omega)d\omega = PU = PQ$.

Using the definition of $Q$ and $P$, the FOC changes into $q(\omega) = \frac{Q}{\lambda(\omega)} \left[\frac{p(\omega)}{\lambda(\omega)P}\right]^{-\sigma}$. Take this into definition of $U$: $U^{\rho} = \int_{\omega\in\Omega} (\lambda(\omega)q(\omega))^{\rho} \, d\omega = \int_{\omega\in\Omega} \left( Q \left[\frac{p(\omega)}{\lambda(\omega)P}\right]^{-\sigma} \right)^{\rho} d\omega$

Figure A4: Percentage of Sales Produced by the Main Product in WBES




Source: World Bank Enterprise Survey China Manufacturing Module 2012.

and we have the expression for aggregate price $P = \left[\int_{\omega\in\Omega}\left(\frac{p(\omega)}{\lambda(\omega)}\right)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}}$. The aggregate price is a quality adjusted price index.

### B.3 The Product Strategy

We prove that Assumption 3.2 implies $\varphi_{hl,1} < \varphi_{hl,2}$, $\varphi_{hh,1} < \varphi_{hh,2}$ and $\varphi_{ll,1} < \varphi_{ll,2}$.

For firms with appeal shock $(a_h, a_l)$, using the cutoff equations (16) and (17), we have $\left(\frac{a_h\varphi_{hl,1}}{\xi a_l\varphi_{hl,2}}\right)^{\beta} = \frac{f_0+f_1}{f_2}$.

By Assumption 3.2 , $\xi \in (0,1)$ and $\frac{\sigma-1}{1-\alpha(\sigma-1)} > 1$, we have $\left(\frac{a_h}{\xi a_l}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} > \left(\frac{a_h}{a_l}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} > \frac{f_0+f_1}{f_2} = \left(\frac{a_h\varphi_{hl,1}}{\xi a_l\varphi_{hl,2}}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}$. This implies $\frac{\varphi_{hl,1}}{\varphi_{hl,2}} < 1$.

Similar argument applies to firms with appeal shock $(a_h, a_h)$ and $(a_l, a_l)$. From $\left(\frac{1}{\xi}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} > \frac{f_0+f_1}{f_2} = \left(\frac{\varphi_{hh,1}}{\xi\varphi_{hh,2}}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}$, we have $\frac{\varphi_{hh,1}}{\varphi_{hh,2}} < 1$.

### B.3 Average Profit and Average Revenue

We use firms with appeal shock $(a_h, a_l)$ under tax as illustrative example.

From the individual revenue function (12), we have the average revenue for firms producing one product, $\bar{r}^t_{hl,1} = (1-\tau)^{\frac{(1+\alpha)(\sigma-1)}{1-\alpha(\sigma-1)}} B\int_{\varphi^{*t}_{hl,1}}^{\varphi^{*t}_{hl,2}}(a_h\varphi)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}\frac{g(\varphi)}{G(\varphi^{**}_{hl,2})-G(\varphi^{**}_{hl,1})}d\varphi$.

Applying the average revenue to the average profit equation, we have $\bar{\pi}^t_{hl,1} = \kappa(1-\tau)\bar{r}^t_{hl,1} - f_0 - f_1 = \kappa(1-\tau)^{\frac{\sigma}{1-\alpha(\sigma-1)}}B\left(\tilde{\varphi}^{*t}_{hl,1}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} - f_0 - f_1$. The second equality substitutes in the definition of average productivity.

Next we transform the average profit expression and utilize the zero profit condition. This implies $\bar{\pi}^t_{hl,1} = \kappa(1-\tau)^{\frac{\sigma}{1-\alpha(\sigma-1)}}B\left(a_h\varphi^{*t}_{hl,1}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}\left(\frac{\tilde{\varphi}^t_{hl,1}}{a_h\varphi^{*t}_{hl,1}}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} - f_0 - f_1 = \left[\left(\frac{\tilde{\varphi}^t_{hl,1}}{a_h\varphi^{*t}_{hl,1}}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} - 1\right](f_0+f_1)$

Similar argument applies to firms with $(a_h, a_l)$ producing two products. The average revenue is

$$\bar{r}^t_{hl,2} = (1-\tau_t)^{\frac{(1+\alpha)(\sigma-1)}{1-\alpha(\sigma-1)}} B\int_{\varphi^{*t}_{hl,2}}^{\infty}\left(a_h^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} + (\xi a_l)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}\right)\varphi^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}\frac{g(\varphi)}{1-G(\varphi^{**}_{hl,2})}d\varphi$$

and the average profit is

$$
\begin{aligned}
\bar{\pi}^t_{hl,2} &= \kappa(1-\tau)\bar{r}^t_{hl,2} - f_0 - f_1 - f_2 = \kappa(1-\tau)^{\frac{\sigma}{1-\alpha(\sigma-1)}} B \left(\tilde{\varphi}^{*t}_{hl,2}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} - f_0 - f_1 - f_2 \\
&= \kappa(1-\tau)^{\frac{\sigma}{1-\alpha(\sigma-1)}} B \left(a_h \varphi^{*t}_{hl,1}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} \left(\frac{\tilde{\varphi}^t_{hl,2}}{a_h \varphi^{*t}_{hl,1}}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} - f_0 - f_1 - f_2 \\
&= \left[\frac{f_0 + f_1}{f_0 + f_1 + f_2} \left(\frac{\tilde{\varphi}^t_{hl,2}}{a_h \varphi^{*t}_{hl,1}}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} - 1\right] (f_0 + f_1 + f_2)
\end{aligned}
$$

### B.4 Counterfactual Experiments

We use the following counterfactual experiments to understand the three margins in amplifying the distortions.

**PF-FF Experiment** In this experiment, the distortions are removed. However, we keep both the cutoffs of product choices for each incumbent (Product Fixed) and the aggregate measure of firms (Firm Fixed) same as in the distorted economy.

Under these assumptions, firms set the price and quantity to maximize the variable profits, taking the aggregate prices as given. The skilled and unskilled labor markets are clear under new aggregate prices.

Given the distribution of active firms and their associated quality shocks and productivities, the aggregate price can be computed as $\left(P^{1-\sigma}\right)^{\frac{1}{1-\alpha(\sigma-1)}} = D \int (a_\omega \varphi)^{\frac{(\sigma-1)}{1-\alpha(\sigma-1)}} d\omega$ where $D = (\rho)^{\frac{(\sigma-1)}{1-\alpha(\sigma-1)}} \left[\frac{\alpha(1-\frac{1}{\sigma})}{w_h} R\right]^{\frac{\alpha(\sigma-1)}{1-\alpha(\sigma-1)}}$. Taking the aggregate demand for skilled labor, $w_h H = \alpha(1-\frac{1}{\sigma})R$ into the expression of D, we have $P^{-b} = \left(P^{1-\sigma}\right)^{\frac{1}{1-\alpha(\sigma-1)}} = (\rho H^\alpha)^{\frac{(\sigma-1)}{1-\alpha(\sigma-1)}} \int_{\omega\in\Omega} (a_\omega \varphi)^{\frac{(\sigma-1)}{1-\alpha(\sigma-1)}} d\omega$. This aggregate price can be computed without a value of $w_h$.

We next calculate the aggregate unskilled labor used in production. The stationary equilibrium requires $p_{in} M_e = \delta M$, and the entry cost is paid by $w_l L_e = w_l M_e f_e$. Together, the total entry cost is $w_l L_e = \frac{\delta M}{p_{in}} w_l f_e$. Therefore, the unskilled labor used in production $L_p = L - L_e = L - \frac{\delta M}{p_{in}} f_e$.

We can also aggregate the demand for the unskilled worker from firm level, $l_i = a_i^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} Q^{\frac{1}{1-\alpha(\sigma-1)}} P^{\frac{\sigma}{1-\alpha(\sigma-1)}} \left(\frac{\alpha(1-\frac{1}{\sigma})}{w_h}\right)^{\frac{\alpha(\sigma-1)}{1-\alpha(\sigma-1)}} \left(\frac{w_l}{\rho\varphi_i}\right)^{\frac{\alpha(\sigma-1)-\sigma}{1-\alpha(\sigma-1)}} \varphi_i^{-1}$. Aggregate this firm level demand,

$$L_p = \int l_i di = Q^{\frac{1}{1-\alpha(\sigma-1)}} P^{\frac{\sigma}{1-\alpha(\sigma-1)}} \left(\frac{\alpha(1-\frac{1}{\sigma})}{w_h}\right)^{\alpha b} \rho^{-\frac{\alpha(\sigma-1)-\sigma}{1-\alpha(\sigma-1)}} \int a_i^b \varphi_i^b di$$

From the aggregate demand of skilled labor, $w_h H = \alpha(1-\frac{1}{\sigma})R$, we have link $Q = \frac{w_h H}{\alpha(1-\frac{1}{\sigma})P}$ to the unskilled wage. Take this into the aggregate demand of unskilled labor, we can find the equilibrium wage $w_h$ that clears both skilled and unskilled labor markets.

**PF Experiment** In this experiment, the distortions are removed and the firm can enter freely. However, the product cutoffs are held as the same as in the distorted economy. Some firms will earn negative profit and stay in the equilibrium.

The entrants will enter until $p_{in}\frac{\bar{\pi}}{\delta} = w_l f_e$. At a stationary equilibrium, we have $w_l L_e = M\bar{\pi} = \Pi$. Therefore, the revenue is $R = PQ = w_l L_p + w_h H + \Pi = w_l L + w_h H$. Combined it with the aggregate demand function of skilled labor, $w_h H = \alpha(1-\frac{1}{\sigma})R$, we have $R = \frac{w_l L}{1-\alpha(1-\frac{1}{\sigma})}$ and $w_h H = \frac{\alpha(1-\frac{1}{\sigma})}{1-\alpha(1-\frac{1}{\sigma})} w_l L$.

The fixed product cutoffs affect the welfare through the its effect on the aggregate price:

$$(P)^{-\frac{\sigma-1}{1-\alpha(\sigma-1)}} = (\rho H^\alpha)^{\frac{(\sigma-1)}{1-\alpha(\sigma-1)}} \int_{\omega\in\Omega} (a_\omega \varphi)^{\frac{(\sigma-1)}{1-\alpha(\sigma-1)}} d\omega.$$

For a guess for $M$, we can compute the associated aggregate price $P$. Then we compute the aggregate revenue by summing up firm level revenue with the help of $B = \left(\rho^{\sigma-1} R P^{\sigma-1}\right)^{\frac{1}{1-\alpha(\sigma-1)}} \left[\frac{\alpha(1-\frac{1}{\sigma})}{w_h}\right]^{\frac{\alpha(\sigma-1)}{1-\alpha(\sigma-1)}}$. Next, we use the free entry condition to check whether the implied aggregate profit from the guessed $M$ satisfies the the free entry condition, $p_{in}\frac{\Pi}{M\delta} = w_l f_e$.

**FF Experiment** In this experiment, the distortions are removed. The distribution of firms, in terms of its measure, the distribution of productivity and quality shocks, is same as in the distorted economy. However, the product choice for each firm can be re-optimized. Therefore, the firm will produce unless it is profitable and some incumbents will not produce in the new economy.

We solve the new equilibrium by guessing the cutoff $\varphi^*_{hh,1}$. Given this guess, we can compute the value of $B = \frac{w_l(f_0+f_1)}{\kappa\left(a_h\varphi^*_{hh,1}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}}$, and find out all other cutoffs by using the zero profit conditions. $\kappa B\left(a_h\varphi^*_{hh,1}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} = w_l(f_0+f_1)$, $\kappa B\left(a_h\varphi^*_{hl,1}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} = w_l(f_0+f_1)$ , $\kappa B\left(a_l\varphi^*_{ll,1}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} = w_l(f_0+f_1)$,

$\kappa B\left(\xi a_h\varphi^*_{hh,2}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} = w_l f_2$, $\kappa B\left(\xi a_l\varphi^*_{hl,2}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} = w_l f_2$ and $\kappa B\left(\xi a_l\varphi^*_{ll,2}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} = w_l f_2$.

With these cutoffs, we can compute the demand for the unskilled labor in firm level $l_i = \rho B a_i^b\varphi_i^b$ and associated aggregate demand, $L_p = \int l_i di = \rho B\int a_i^b\varphi_i^b di$. We check if this number equals the unskilled labor available in production $L_p = L - L_e = L - \frac{\delta M}{p_{in}}f_e$ and find out the right guess of $\varphi^*_{hh,1}$.

### B.5 The continuous distortion case

The distortion is assumed to be continuous in the productivity $\varphi$. That is, $1-\tau(\varphi) = \varphi^{-\gamma}$ where $\gamma \in (0,1)$.

For active firms, the optimal pricing rule is $p_i = \frac{w_l}{(1-\tau)\rho\varphi} = \frac{w_l}{\rho\varphi^{1-\gamma}}$. Using the inverse demand function, its optimal quantity and quality in production are $q_i = \varphi_i^{\frac{\sigma(1-\gamma)-\alpha(\sigma-1)}{1-\alpha(\sigma-1)}} a_i^{\frac{\sigma-1}{1-\alpha(\sigma-1)}} Q^{\frac{1}{1-\alpha(\sigma-1)}} P^{\frac{\sigma}{1-\alpha(\sigma-1)}} \left(\frac{\alpha(1-\frac{1}{\sigma})}{w_h}\right)^{\frac{\alpha(\sigma-1)}{1-\alpha(\sigma-1)}} \left(\frac{w_l}{\rho}\right)^{\frac{\alpha(\sigma-1)-\sigma}{1-\alpha(\sigma-1)}}$ and $\lambda_i = \varphi_i^{\frac{\alpha\sigma(1-\gamma)-\alpha}{1-\alpha(\sigma-1)}} a_i^{\frac{1}{1-\alpha(\sigma-1)}} Q^{\frac{\alpha}{1-\alpha(\sigma-1)}} P^{\frac{\alpha\sigma}{1-\alpha(\sigma-1)}} \left(\frac{\alpha(1-\frac{1}{\sigma})}{w_h}\right)^{\frac{\alpha}{1-\alpha(\sigma-1)}} \left(\frac{w_l}{\rho}\right)^{\frac{-\alpha(\sigma-1)}{1-\alpha(\sigma-1)}}$

The product strategies are derived from the zero profit conditions. For firms with $(a_h, a_h)$ and $(a_h, a_l)$, they will at least produce one product if $\varphi \geq \varphi^*_{hh,1}$, where the cutoff comes from the zero profit condition $\kappa B\left(a_h\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}\left(\varphi^*_{hh,1}\right)^{\frac{\sigma(1-\gamma)-1}{1-\alpha(\sigma-1)}} = w_L\left(f_0+f_1\right)$. Firms with $(a_h, a_h)$ will produce two products if $\varphi \geq \varphi^*_{hh,2}$, where

$\kappa B\left(\xi a_h\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}\left(\varphi^*_{hh,2}\right)^{\frac{\sigma(1-\gamma)-1}{1-\alpha(\sigma-1)}}=w_L f_2$. Firms with $(a_h,a_l)$ and $(a_l,a_l)$ will produce two products if $\varphi\geq\varphi^*_{hl,2}=\varphi^*_{ll,2}$, where $\kappa B\left(\xi a_l\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}\left(\varphi^*_{hl,2}\right)^{\frac{\sigma(1-\gamma)-1}{1-\alpha(\sigma-1)}}=w_L f_2$. Lastly, two products will be produced for firms with $(a_l,a_l)$ if $\varphi\geq\varphi^*_{ll,1}$, where $\kappa B\left(a_l\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}\left(\varphi^*_{ll,1}\right)^{\frac{\sigma(1-\gamma)-1}{1-\alpha(\sigma-1)}}=w_L\left(f_0+f_1\right)$.

For the firms $(a_h,a_l)$ producing two products, the average revenue is

$$\begin{aligned}
\bar{\pi}^t_{hl,2} &= \kappa B\int_{\varphi^{*t}_{hl,2}}^{\infty}\left(a_h^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}+\left(\xi a_l\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}\right)\varphi^{\frac{\sigma(1-\gamma)-1}{1-\alpha(\sigma-1)}}\frac{g(\varphi)}{1-G(\varphi^{**}_{hl})}d\varphi-f_0-f_1-f_2\\
&= \kappa B\left(\tilde{\varphi}^{*t}_{hl,2}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}-f_0-f_1-f_2\\
&= \kappa B a_h^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}\left(\varphi^{*t}_{hl,1}\right)^{\frac{\sigma(1-\gamma)-1}{1-\alpha(\sigma-1)}}\left(\frac{\tilde{\varphi}^t_{hl,2}}{a_h(\varphi^{*t}_{hl,1})^d}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}-f_0-f_1-f_2\\
&= \left[\frac{f_0+f_1}{f_0+f_1+f_2}\left(\frac{\tilde{\varphi}^t_{hl,1}}{a_h(\varphi^{*t}_{hl,1})^d}\right)^{\frac{\sigma-1}{1-\alpha(\sigma-1)}}-1\right]\left(f_0+f_1+f_2\right)
\end{aligned}$$

where $d=\frac{\sigma-1}{\sigma-\alpha(\sigma-1)}$.

The average profit is the sum of two parts $\bar{\pi}_{hl}=\frac{G(\varphi^{**}_{hl})-G(\varphi^{**}_{h})}{1-G(\varphi^{**}_{h})}\bar{\pi}_{hl,1}+\frac{1-G(\varphi^{**}_{hl})}{1-G(\varphi^{**}_{h})}\bar{\pi}_{hl,2}$

Similar calculations for firms with $(a_h,a_h)$ and $(a_l,a_l)$. The cutoffs are related with each other from the zero profit conditions. The average profit expression for the whole economy gives us a relation between average profit $\bar{\pi}$ and cutoff $\varphi^*_{hl,1}$.

We solve the equilibrium with guess and verify. Guessing a $\varphi^*_{hl,1}$, we have cutoffs for all. From the relationship between the average profit and the cutoff, we know the average profit $\bar{\pi}$.

The free entry condition requires: $G(\varphi^*_{hl,1},\Gamma)\bar{\pi}=\delta w_1 f_e$, where $G(\varphi^*_{hl,1},\Gamma)$ is the surviving probability under the systematic distortion $\Gamma$. A right guess of $\varphi^*_{hl,1}$ and the average profit $\bar{\pi}$ will satisfy the free entry condition. Like the equilibrium without distortions, given a particular distortion pattern, the cutoffs and the average profit are pined down separated from other aggregate variables.

Aggregate the firm level demand for human capital, we have

$$\begin{aligned}
w_h H &= \alpha(1-\frac{1}{\sigma})\int(1-\tau)p_i q_i di \\
&= \alpha(1-\frac{1}{\sigma})M\sum_{n=s,t,f}\sum_{k=hh,hl,ll}\left[\frac{M_{k,1}^n}{M}(1-\tau_n)\bar{r}_{k,1}^n+\frac{M_{k,2}^n}{M}(1-\tau_n)\bar{r}_{k,2}^n\right] \\
&= \alpha(1-\frac{1}{\sigma})M\tilde{r}
\end{aligned}$$

where $\tilde{r}$ is the distortion combined average revenue. Without tax rebate, we have $R = M\bar{r} = w_L L + w_h H$. Combine with the revenue equation, we have $M\bar{r} = w_L L + \alpha(1-\frac{1}{\sigma})M\tilde{r}$. Then we can solve the measure of firms $M = \frac{L}{\bar{r}-\alpha(1-\frac{1}{\sigma})\tilde{r}}$ and total revenue $R = M\bar{r}$.